STATISTICAL HORIZONS

Mediation, Moderation, and Conditional Process Analysis

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Upcoming Seminar:

August 17-20, 2021, Remote Seminar

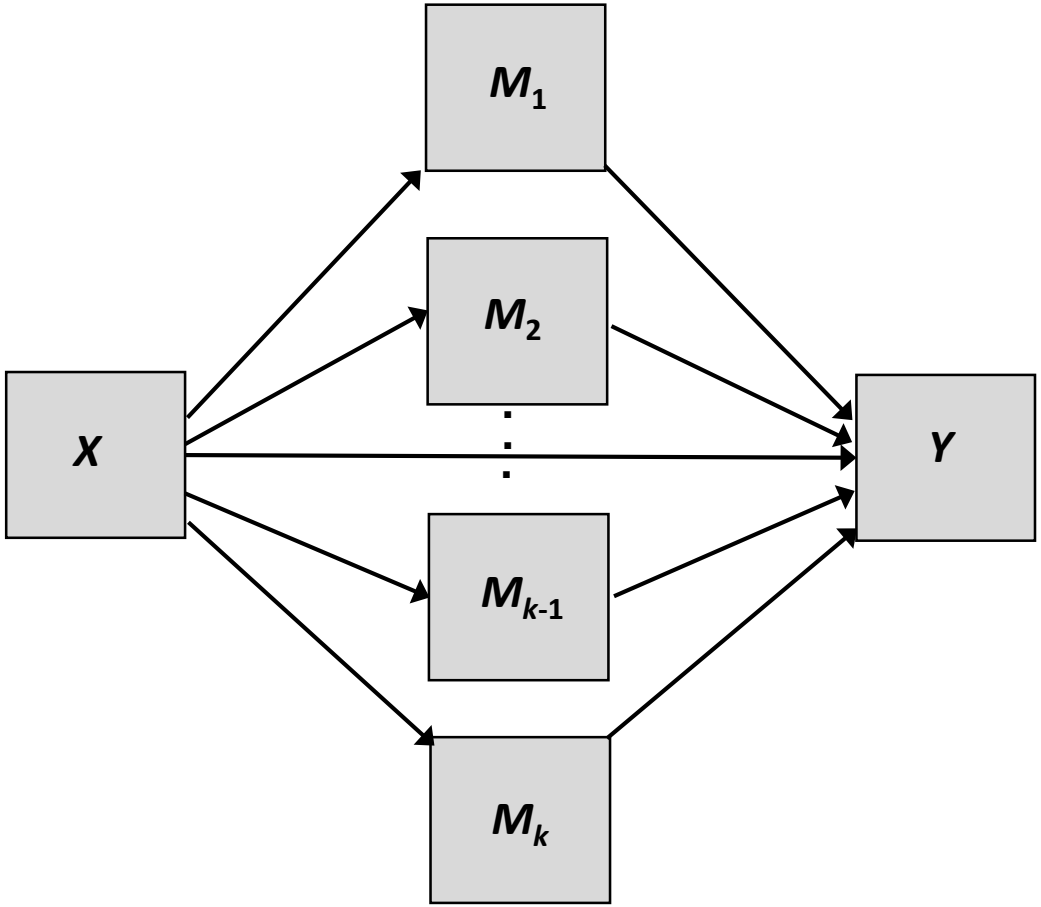
Mediation, Moderation, and Conditional Process Analysis

Instructor: Amanda K. Montoya

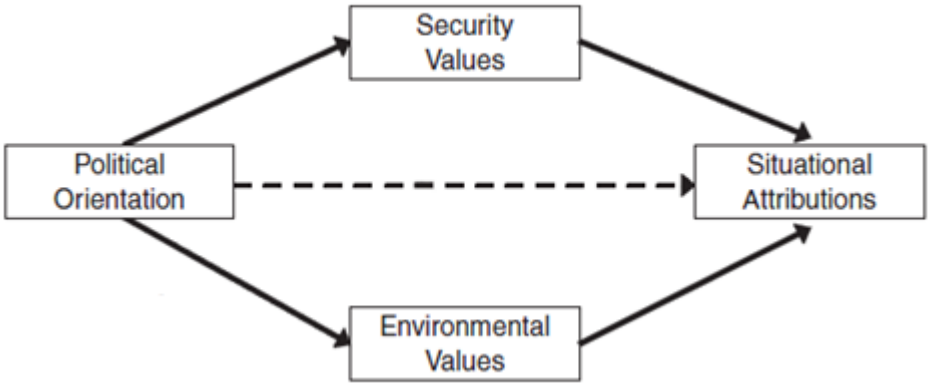
STATISTICAL HORIZONS

Models with More Than One Mediator

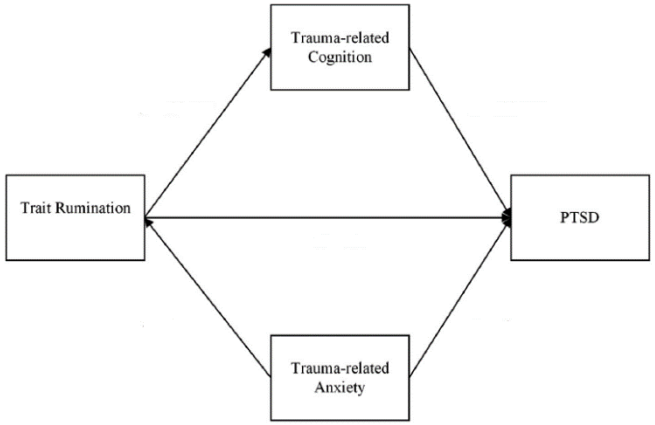
A parallel multiple mediator model



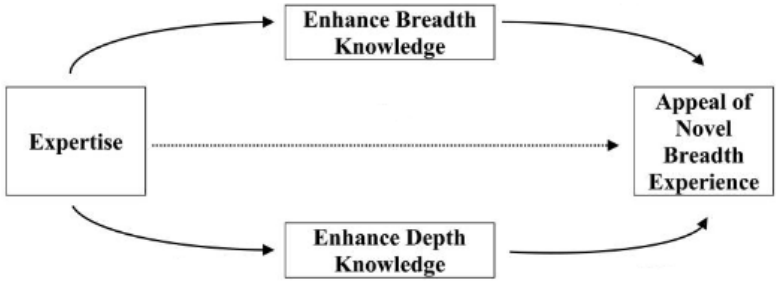
Some examples From the literature with 2 mediators



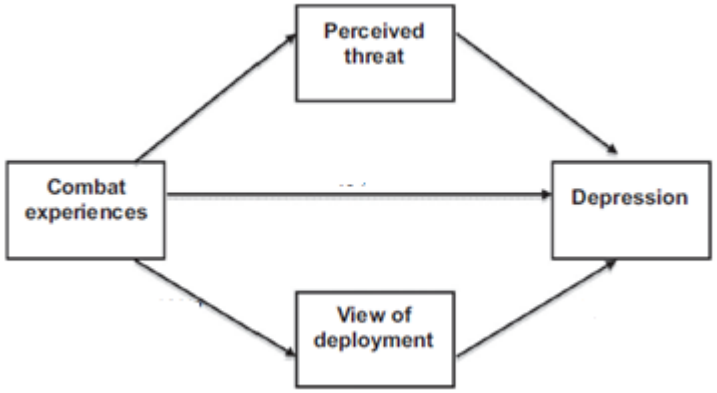
Morgan, G. S., Mullen, E., & Skitka, L. J. (2010). When values and attributions collide: Liberals' and conservatives' values motivate attributions for alleged misdeeds. ***Personality and Social Psychology Bulletin***, 36, 1241-1254.



Spinhoven, P., Penninx, B. W., Krempeniou, A., et al. (2015). Trait rumination predicts onset of post-traumatic stress disorder through trauma-related cognitive appraisals: A 4-year longitudinal study. ***Behaviour Research and Therapy, 71***, 101-109.

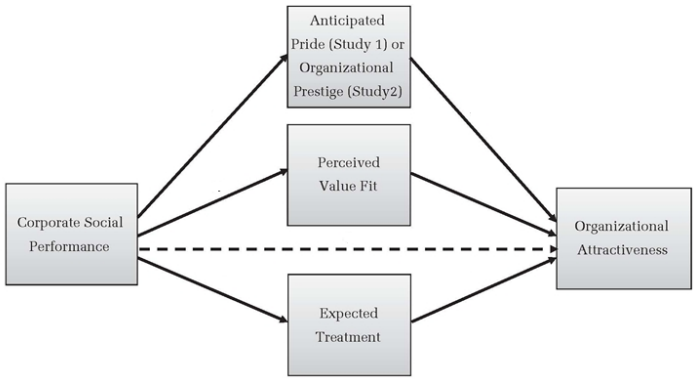


Clarkson, J. J., Janiszewski, C., & Cinelli, M. D. (2013). The desire for consumption knowledge. ***Journal of Consumer Research,*** *39,* 1313-1329.

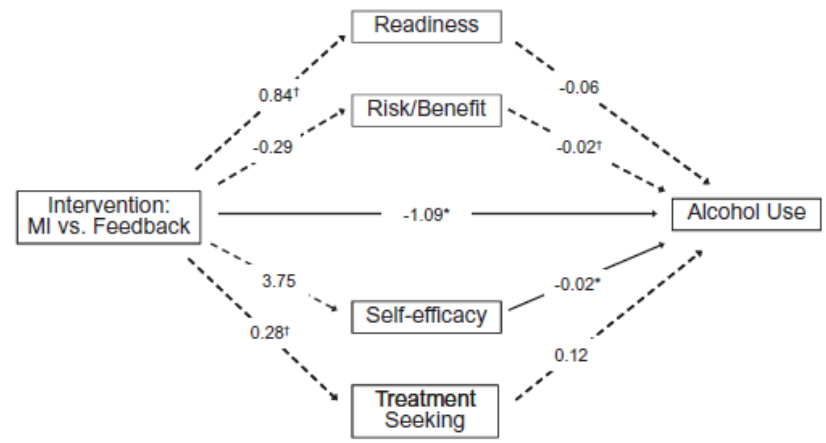


Pitts, B. L., & Safer, M. A. (2016). Retrospective appraisals mediate the effect of combat experiences on PTS and depression symptoms in U.S. Army medics***. Journal of Traumatic Stress, 29***, 65-71.

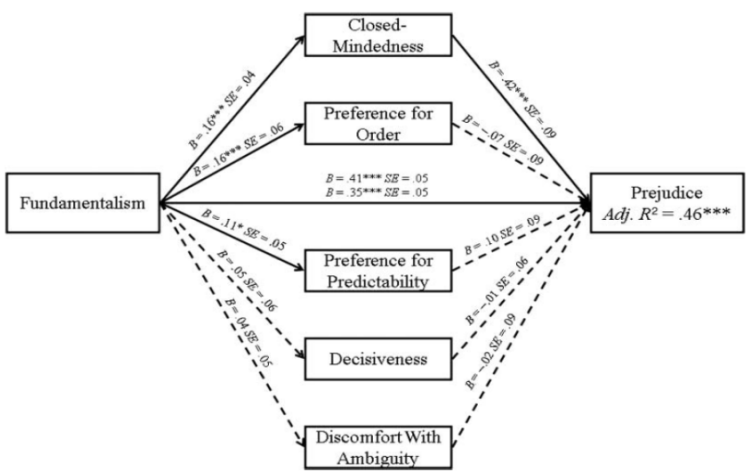
Some examples from the literature with several mediators



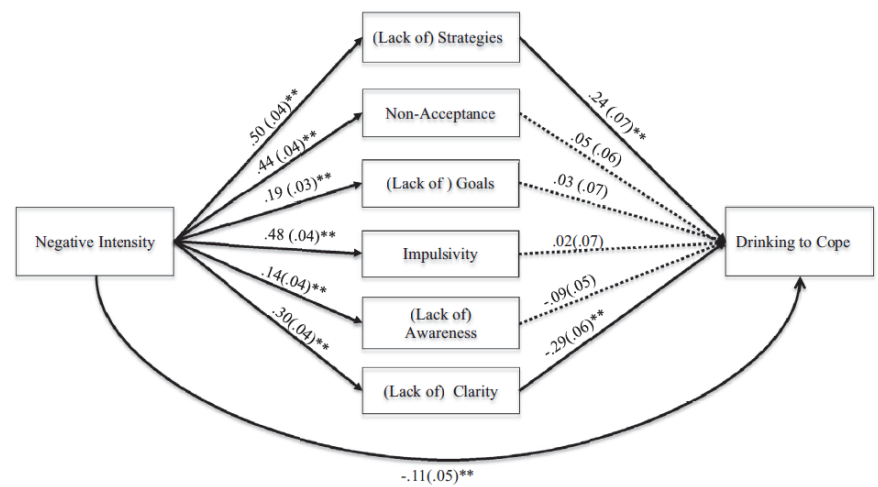
Jones, D. A., Willness, C. R., & Madey, S. (2014). Why are job seekers attracted by corporate social performance? Experimental and field tests of three signal-based mechanisms. ***Academy of Management Journal, 57***, 383-404.



Barnett, N. P., Apodaca, T. R., et al. (2010). Moderators and mediators of two brief interventions for alcohol in the emergency department. *Addiction, 105*, 452-465.



Brandt, M. J., & Reyna, C. (2010). The role of prejudice and the need for closure in religious fundamentalism. ***Personality and Social Psychology Bulletin,*** *36*, 715-725.



Veilleux, J. C.,Skinner, K. D., Reese, E. D., & Shaver, J. A. (2014). Negative affect intensity influences drinking to cope through facets of emotion dysregulation. ***Personality and Individual Differences,*** *49*, 96-101.

Why estimate such a model?

- Many causal effects probably operate through multiple mechanisms simultaneously. Better to estimate a model **consistent with such real-world** complexities.

- If your proposed mediator is correlated with the real mediator but not caused by the independent variable, a model with only your proposed mediator in it will be a **misspecification** and will potentially misattribute the process to your proposed mediator rather than the real mediator—"epiphenomenality."

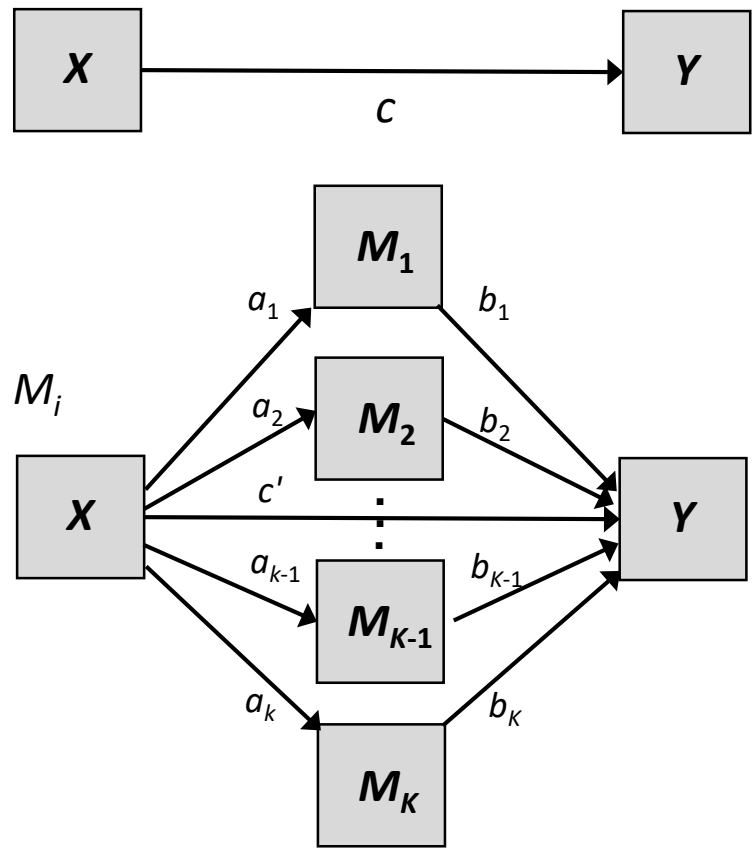
- Different theories may postulate different mediators as mechanisms. Including them all in a model simultaneously allows for a formal statistical comparison of indirect effects **representing different theoretical mechanisms**.

Path Analysis: Total, Direct, and Indirect Effects

$$\hat{Y} = c_0 + cX$$

$$\widehat{M_j} = a_{0j} + a_j X$$

$$\hat{Y} = c'_0 + c'X + \sum_{j=1}^{K} b_j M_j$$



c = "total effect" of X on Y

$a_j \times b_j$ = "specific indirect effect" of X on Y through M_i

$\Sigma\,(a_j \times b_j)$= "total indirect effect" of X on Y

c' = "direct effect" of X on Y

total effect = direct effect + total indirect effect

$$c \quad = \quad c' \quad + \quad \Sigma\,(a_j \times b_j)$$

total indirect effect = total effect – direct effect

$$\Sigma\,(a_j \times b_j) \quad = \quad c \quad - \quad c'$$

Example: Science

Participants read a syllabus for a computer science class. The syllabus one of two policies: **procollaboration** or **no collaboration**. Participants were randomly assigned to condition.

Participants completed questionnaire (Higher = greater):

(1) interest in the class (this is the primary DV).

(2) how much they felt the class would help them in achieving **communal goals** (helping others, working with others)

(3) how **difficult** they expected the class to be.

Girls who Code

Question: Does group work in computer science classes increase interest in the class indirectly through perceived communal goal fulfillment, through class difficulty, or both?

Would people who read about the procollaboration policy think the class is more communal and would that communality then predict greater interest? Would the procollaboration policy make students think the course is easier, and this would increase interest?

The data: Science

ProNo	comm	diff	interest
1.00	5.20	5	6.0
1.00	1.00	4	2.2
1.00	4.00	4	2.5
1.00	4.00	2	3.5
1.00	7.00	7	7.0
1.00	6.00	1	6.0
1.00	4.00	7	2.7
1.00	3.40	6	4.2
1.00	4.20	3	3.5
1.00	4.60	6	1.5
1.00	4.20	5	2.0
1.00	4.40	5	1.0
1.00	5.40	3	7.0
1.00	5.00	4	2.2
1.00	4.60	6	2.7
1.00	3.40	4	1.2

```
lata science;
   input Subject Cond sex ProNo comm diff interest;
   datalines;
    106      1      1      1    5.2      5      6
    109      1      1      1      1      4   2.25
    112      1      1      1      4      4    2.5
    114      1      1      1      4      2    3.5
    115      1      1      1      7      7      7
    121      1      1      1      6      1      6
    131      1      1      1      4      7   2.75
    132      1      1      1    3.4      6   4.25
    148      1      1      1    4.2      3    3.5
    161      1      1      1    4.6      6    1.5
    162      1      1      1    4.2      5      2
    164      1      1      1    4.4      5      1
    174      1      1      1    5.4      3      7
    176      1      1      1      5      4   2.25
    177      1      1      1    4.6      6   2.75
    178      1      1      1    3.4      4   1.25
    190      1      1      1    3.8      5   2.75
    206      1      1      1    4.2      6   1.75
    216      1      1      1      7      7      6
    217      1      1      1      5      4   4.75
```

ProNo: Experimental condition (1 = procollaboration, 0 = no collaboration)

interest : interest in class (higher = greater interest)

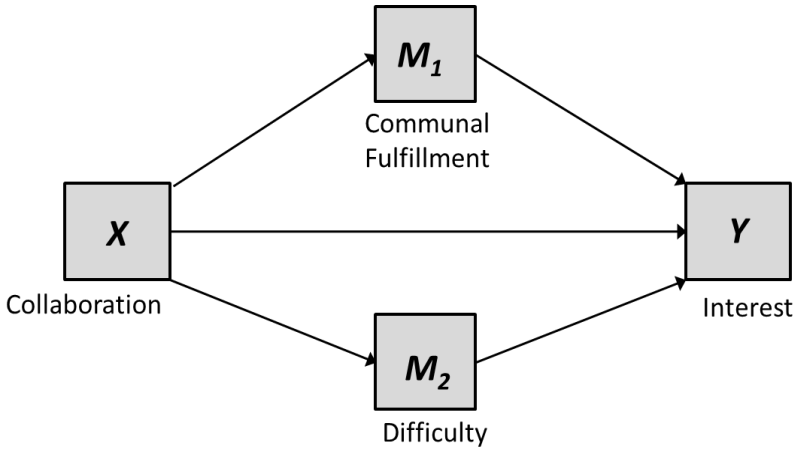
comm: Perceived fulfillment of communal goals (higher = more fulfillment)

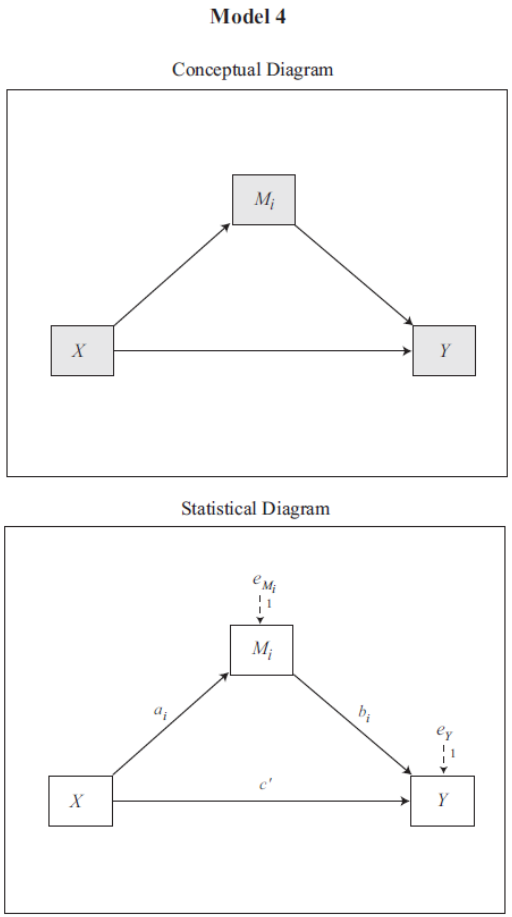
diff: Perceived difficulty of the class (higher = more difficult)

gender: 1 = Male, 2 = Female

Estimation and inference using PROCESS

PROCESS model 4 is used for the parallel multiple mediator model.





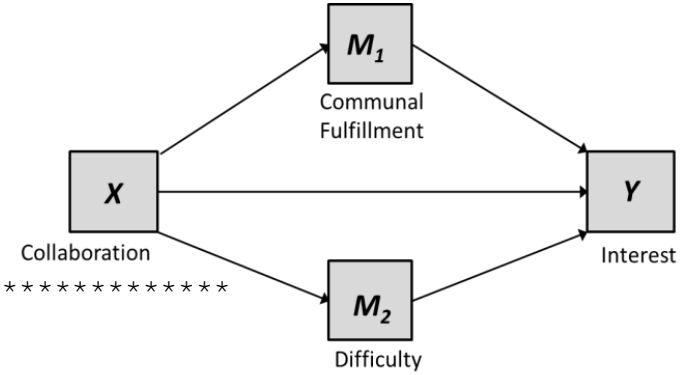
Up to 10 mediators can be listed in the "m =" list. Order does not matter.

```
process y=interest/x=ProNo/m=comm diff/total=1/boot=10000/model=4/normal=1/contrast=1.
```

```
%process (data=science,y=interest,x=ProNo,m=comm diff,total=1,boot=10000,model=4,
     normal=1,contrast=1);
```

PROCESS output

Output E




```
Model  : 4
    Y  : interest
    X  : ProNo
   M1  : comm
   M2  : diff
Sample  Size:  232
**************************************************************************
OUTCOME VARIABLE:
 comm
```

$$\widehat{M_1} = 3.12 + 0.78X$$

```
Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .3031      .0919     1.5279    23.2670     1.0000   230.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.1160      .1133    27.4994      .0000     2.8927     3.3392
ProNo         .7831      .1624     4.8236      .0000      .4632     1.1030
```

a_1 path

```
**************************************************************************
OUTCOME VARIABLE:
 diff
```

$$\widehat{M_2} = 4.94 - 0.15X$$

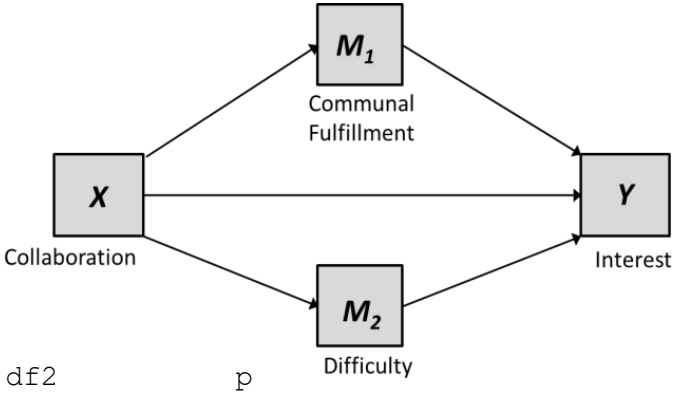
```
Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .0594      .0035     1.6760      .8155     1.0000   230.0000      .3674

Model
              coeff         se          t          p       LLCI       ULCI
constant     4.9412      .1187    41.6352      .0000     4.7073     5.1750
ProNo       -.1536      .1700     -.9031      .3674     -.4886      .1815
```

a_2 path

PROCESS output

Output E




```
OUTCOME VARIABLE:
 interest

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4418      .1952     1.9659    18.4348     3.0000   228.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant      .4898      .4618     1.0607      .2899     -.4201     1.3996
ProNo        -.0895      .1933     -.4630      .6438     -.4705      .2914
comm          .5367      .0752     7.1418      .0000      .3886      .6848
diff          .1364      .0718     1.9008      .0586     -.0050      .2778

************************* TOTAL EFFECT MODEL ***************************
OUTCOME VARIABLE:
 interest

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .1000      .0100     2.3974     2.3217     1.0000   230.0000      .1290

Model
              coeff         se          t          p       LLCI       ULCI
constant     2.8361      .1419    19.9817      .0000     2.5565     3.1158
ProNo         .3099      .2034     1.5237      .1290     -.0908      .7106
```

$$\hat{Y} = 0.49 - 0.09X + 0.54M_1 + 0.14M_2$$

c' path

b_1 path

b_2 path

$$\hat{Y} = 2.84 + 0.31X$$

c path

PROCESS output

Output E

```
************** TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y **************

Total effect of X on Y
     Effect         se          t          p       LLCI       ULCI
      .3099      .2034     1.5237      .1290     -.0908      .7106

Direct effect of X on Y
     Effect         se          t          p       LLCI       ULCI
     -.0895      .1933     -.4630      .6438     -.4705      .2914

Indirect effect(s) of X on Y:
           Effect     BootSE   BootLLCI   BootULCI
TOTAL       .3994      .1135      .1950      .6399
comm        .4203      .1128      .2171      .6594
diff       -.0209      .0282     -.0858      .0280
(C1)        .4413      .1189      .2251      .6927

   Normal theory test for indirect effect(s):
         Effect         se          Z          p
comm      .4203      .1059     3.9706      .0001
diff     -.0209      .0284     -.7367      .4613

Specific indirect effect contrast definition(s):
(C1)           comm       minus    diff
```

c path

c′ path

$a_1b_1 + a_2b_2$ with bootstrap CI

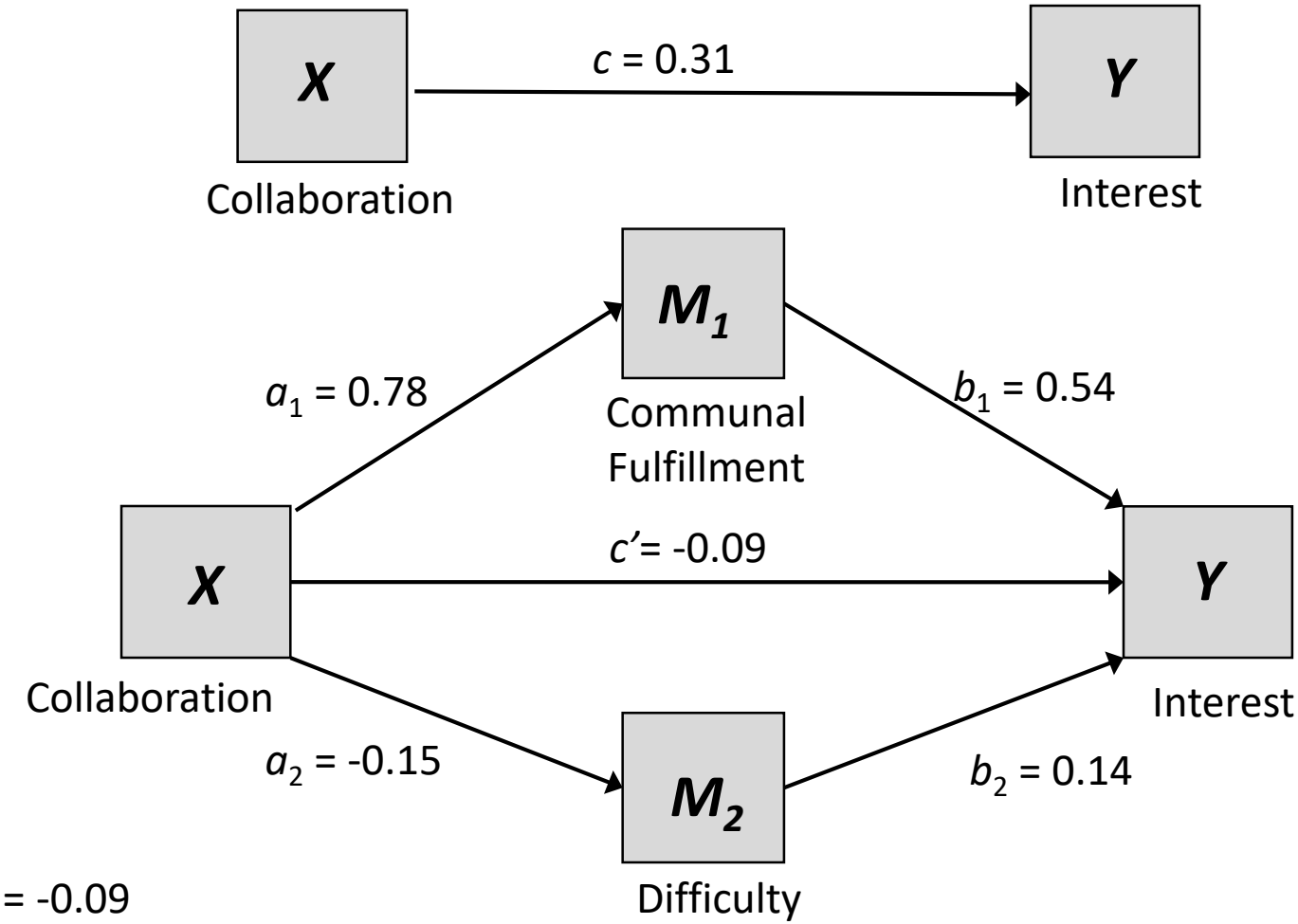
a_1b_1 with bootstrap CI

a_2b_2 with bootstrap CI

Sobel tests (less trustworthy than bootstrap CIs)

The data are consistent with the claim that group work influences interest indirectly through communal goal fulfillment controlling for difficulty (0.420; 95% CI = 0.217 to 0.659) but not through difficulty controlling for goal fulfillment (-0.0209; 95% CI = -0.086 to 0.028).

Example: Science



Direct effect = -0.09
Specific indirect effect via Comm: 0.78(0.54) = 0.42
Specific indirect effect via Diff: -0.15(0.14) = -0.02
Total indirect effect = 0.42 – 0.02 = 0.40
Total effect = -0.09+ .42 – 0.02= 0.31

Things to consider

(1) In a multiple mediator model, the specific indirect effect through M_k quantifies the component of the total indirect effect that is unique to M_k. Each specific indirect effect is estimated **controlling for all other mediators.**

M_k may function as a mediator variable when considered in isolation but not when considered with other mediator variables in the same model. If the intervening variables are highly intercorrelated, they can "cancel out" each others' effects.

(2) It is possible for a total indirect effect to be not detectably different from zero even when one or more specific indirect effects is.

total indirect effect = sum of specific indirect effects

$$\Sigma (a_i b_i) \quad = \quad a_1 b_1 + a_2 b_2 + \ldots a_k b_k.$$

Scenario (a): A single large specific indirect combined with several tiny ones.

Scenario (b): Specific indirect effects that have different signs and add to near zero.

In multiple mediator models, the total indirect effect is not always of great interest.

Comparing specific indirect effects

Indirect effects quantify how *Y* changes as *X* changes by one unit through a mediator. **They are free of the scale of measurement of the mediators**. So in multiple mediator models, indirect effects linking the same *X* to the same *Y* are directly comparable even If the mediators are measured on different scales. We can statistically compare them if so desired. No standardization or other arithmetic gymnastics is required.

Approach #1: Calculate the ratio of the difference between the indirect effect through M_i and the indirect effect through M_j to its standard error. Assuming a normally distributed sampling distribution of the difference, a *p*-value for the null hypothesis that the difference equals zero can be derived from the standard normal distribution.

$$Z = \frac{a_i b_i - a_j b_j}{se_{a_i b_i - a_j b_j}}$$

Approach #2: Bootstrap a confidence interval for the $a_i b_i - a_j b_j$ and ascertain whether 0 is in the confidence interval as a pseudo null hypothesis test that the difference is zero.

PROCESS can generate a bootstrap confidence interval for all possible pairwise comparisons between specific indirect effects

PROCESS output

```
process y=interest/x=prono/m=comm diff/total=1/boot=10000/model=4/normal=1/contrast=1.
```

```
%process (data=science,y=interest,x=prono,m=comm diff,total=1,boot=10000,model=4,
     normal=1,contrast=1);
```

```
Indirect effect(s) of X on Y:
          Effect     BootSE   BootLLCI   BootULCI
TOTAL      .3994      .1135      .1950      .6399
comm       .4203      .1128      .2171      .6594
diff      -.0209      .0282     -.0858      .0280
(C1)       .4413      .1189      .2251      .6927

   Normal theory test for indirect effect(s):
         Effect         se          Z          p
comm      .4203      .1059     3.9706      .0001
diff     -.0209      .0284     -.7367      .4613

Specific indirect effect contrast definition(s):
(C1)           comm      minus    diff
```

$a_1b_1 - a_2b_2$

Output E

The specific indirect effect of collaboration on interest through perceived Communal goal fulfillment is different from the specific indirect effect through perceived class difficulty (difference = 0.441; 95% CI = 0.225 to .6927).