

# Multilevel and Mixed Models

Stephen Vaisey, Ph.D.

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## What are MLMs?

Most broadly, they are models that estimate parameters from data with:

- multiple observations from the same group (e.g., students in classes)
- repeated observations from the same person
  - sometimes called panel data or longitudinal data
  - not the primary focus here but *very similar*

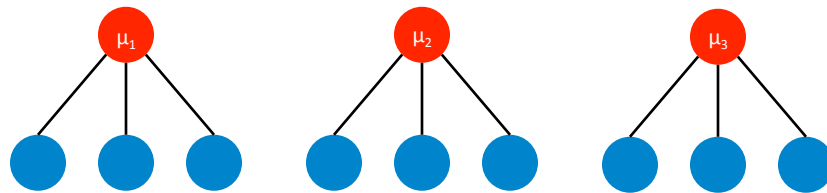
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## Why Use Multilevel Models?

- avoid problems with basic models (good)
- ask more interesting questions (better)

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## Hierarchical DGP



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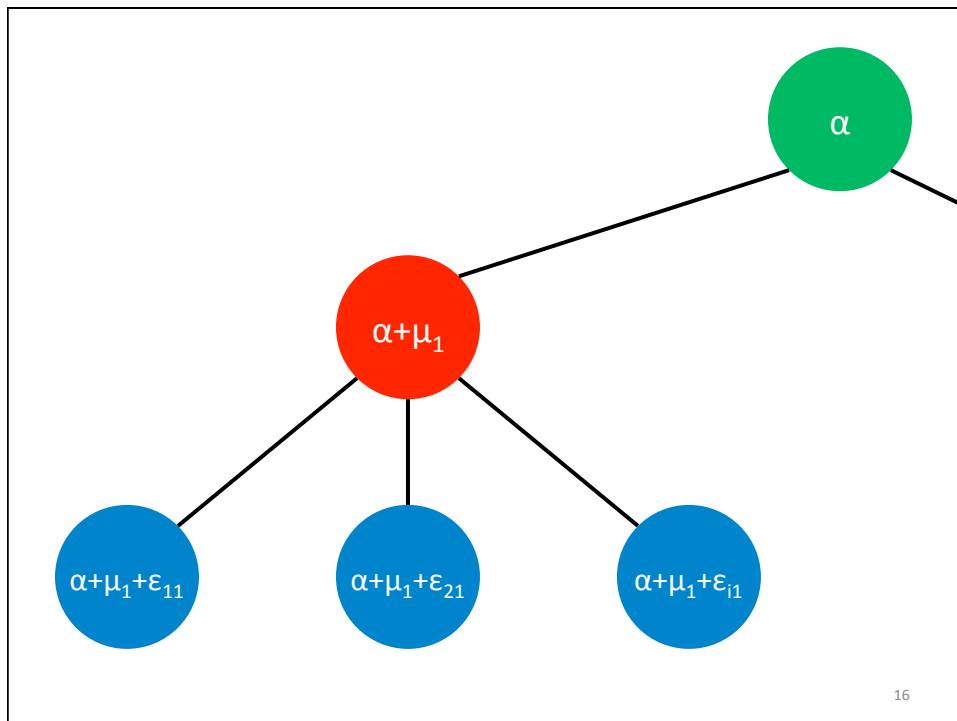
## DGP Simulation Logic

```
set obs 5                // # of clusters
gen u = rnormal(0,1)    // the different  $\mu$  values
expand 10               // how many "people" per cluster
gen y = u + rnormal(0,1) // observed variable
```

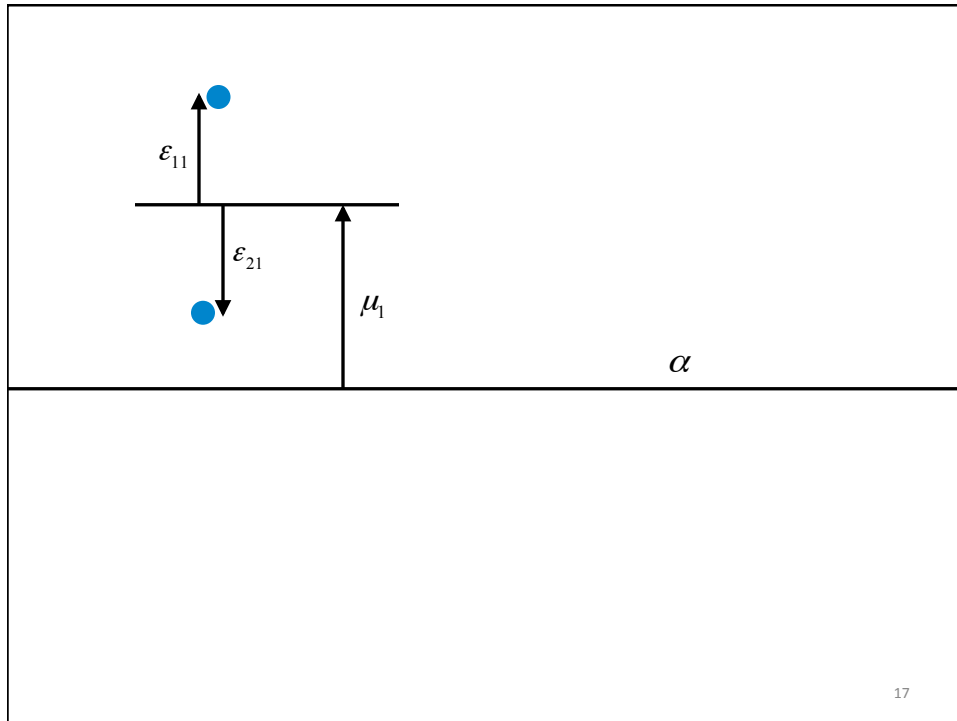
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$$y_{ij} = \alpha + \mu_j + \varepsilon_{ij}$$

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Different notations, same idea

$$y_{ij} = \alpha + \mu_j + \varepsilon_{ij}$$

$$y_{ij} = \mu + \alpha_j + \varepsilon_{ij}$$

$$y_{ij} = \beta + \zeta_j + \varepsilon_{ij}$$

$$y_{ij} = \beta_0 + u_j + e_{ij}$$

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## Main Point

Because the observations are not independent, using only a single error term ( $\varepsilon_i$ ) is not justified. The errors of the individual observations may be correlated, violating a core assumption of OLS regression.

However, if that's your *only* concern, that can be dealt with using clustered SEs.

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## Example: Life Satisfaction in Europe

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## Data

200 person per country subsample of 2014 ESS  
(for faster estimation)

**B20 CARD 13** All things considered, how satisfied are you with your life as a whole nowadays? Please answer using this card, where 0 means extremely<sup>18</sup> dissatisfied and 10 means extremely satisfied.

**Extremely  
dissatisfied**

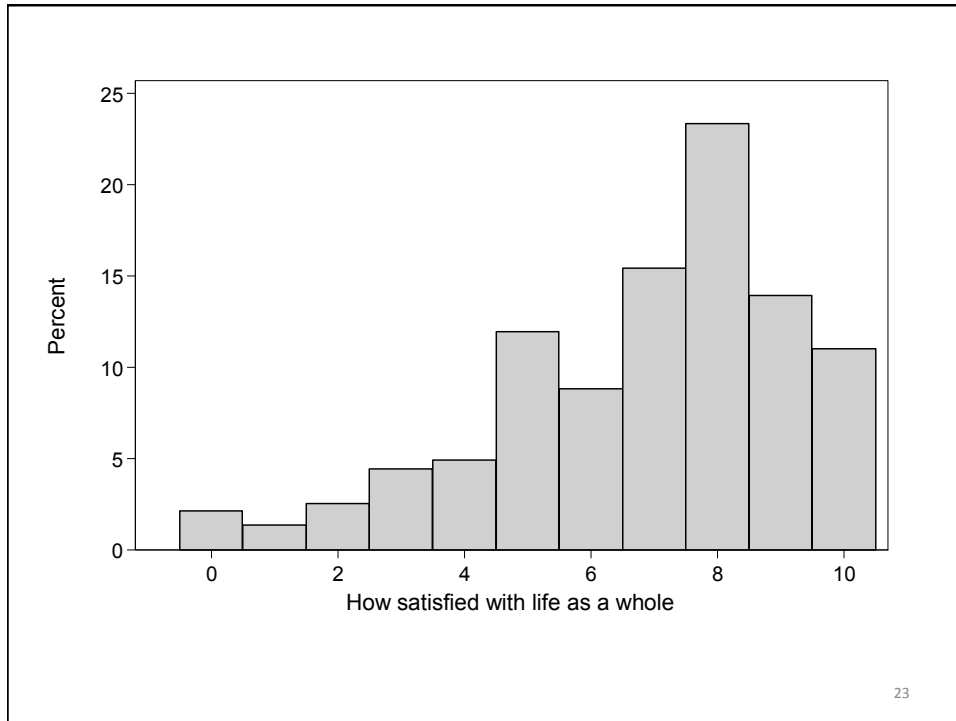
**Extremely  
satisfied**

00 01 02 03 04 05 06 07 08 09 10

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How satisfied with life as a whole	Freq.	Percent	Cum.
Extremely dissatisfied	<b>125</b>	<b>2.16</b>	<b>2.16</b>
1	<b>79</b>	<b>1.36</b>	<b>3.52</b>
2	<b>148</b>	<b>2.55</b>	<b>6.07</b>
3	<b>258</b>	<b>4.45</b>	<b>10.52</b>
4	<b>286</b>	<b>4.93</b>	<b>15.45</b>
5	<b>694</b>	<b>11.97</b>	<b>27.41</b>
6	<b>512</b>	<b>8.83</b>	<b>36.24</b>
7	<b>895</b>	<b>15.43</b>	<b>51.67</b>
8	<b>1,354</b>	<b>23.34</b>	<b>75.02</b>
9	<b>809</b>	<b>13.95</b>	<b>88.97</b>
Extremely satisfied	<b>640</b>	<b>11.03</b>	<b>100.00</b>
Total	<b>5,800</b>	<b>100.00</b>	

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```

. sum stflife, d

```

How satisfied with life as a whole

Percentiles		Smallest		
1%	0	0		
5%	2	0		
10%	3	0	obs	5,800
25%	5	0	Sum of wgt.	5,800
50%	7		Mean	6.783966
75%	8	Largest	Std. Dev.	2.414616
90%	10	10	Variance	5.830369
95%	10	10	Skewness	-.8581936
99%	10	10	Kurtosis	3.29755

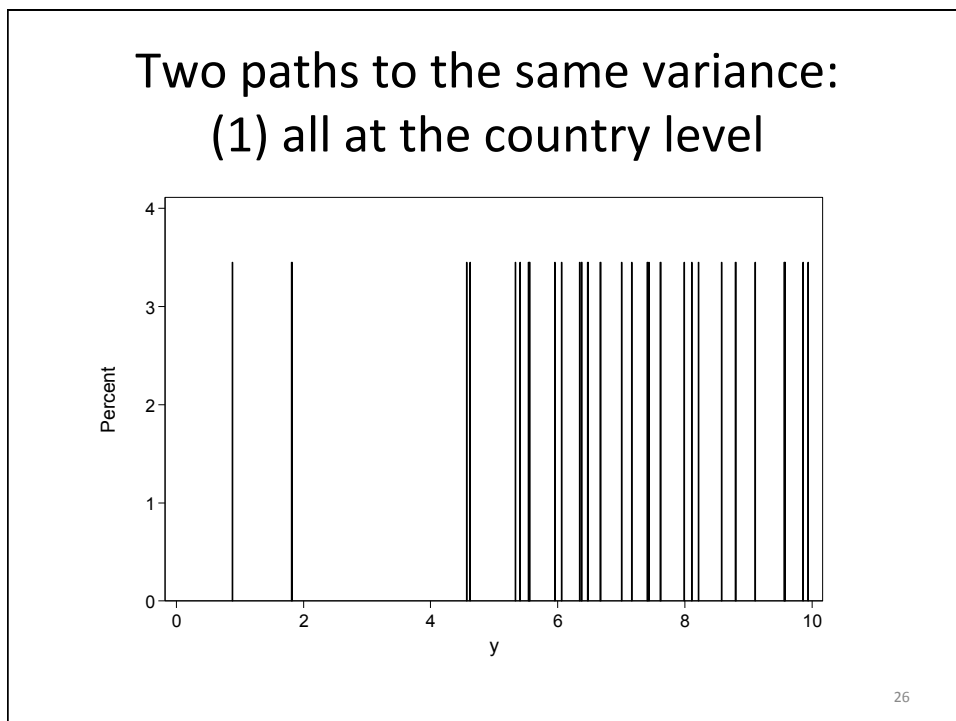
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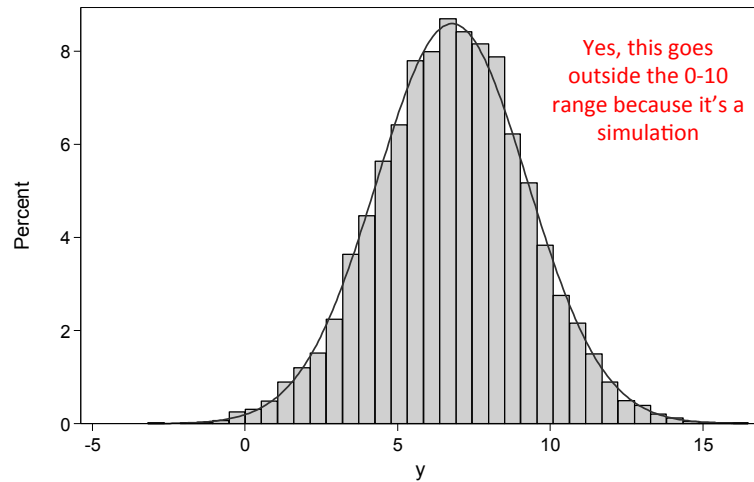
`. tab country`

Country	Freq.	Percent	Cum.
AL	200	3.45	3.45
BE	200	3.45	6.90
BG	200	3.45	10.34
CH	200	3.45	13.79
CY	200	3.45	17.24
CZ	200	3.45	20.69
DE	200	3.45	24.14
DK	200	3.45	27.59
EE	200	3.45	31.03
ES	200	3.45	34.48
FI	200	3.45	37.93
FR	200	3.45	41.38
GB	200	3.45	44.83
HU	200	3.45	48.28
IE	200	3.45	51.72
IL	200	3.45	55.17
IS	200	3.45	58.62
IT	200	3.45	62.07
LT	200	3.45	65.52
NL	200	3.45	68.97
NO	200	3.45	72.41
PL	200	3.45	75.86
PT	200	3.45	79.31
RU	200	3.45	82.76
SE	200	3.45	86.21
SI	200	3.45	89.66
SK	200	3.45	93.10
UA	200	3.45	96.55
XK	200	3.45	100.00
Total	5,800	100.00	

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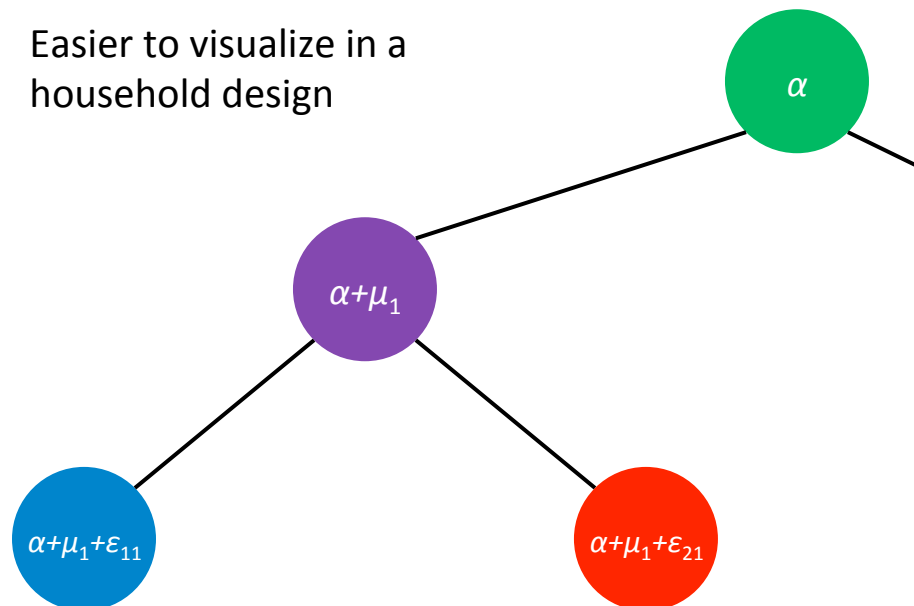


## Two paths to the same variance: (2) all at the individual level



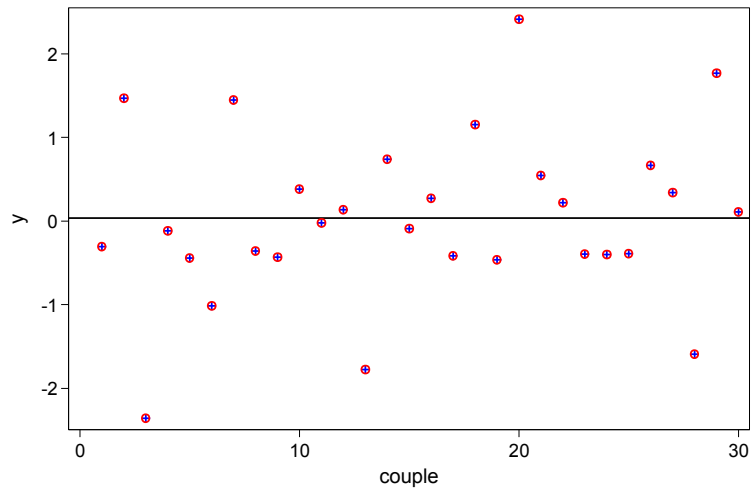
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Easier to visualize in a household design



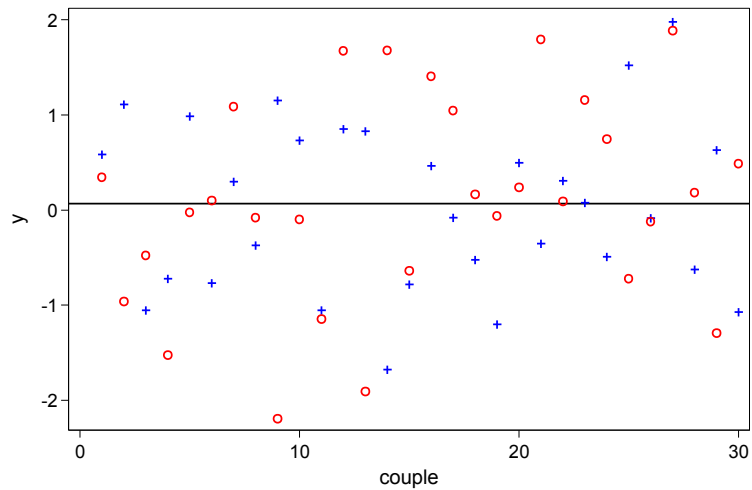
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## All variance at the household level



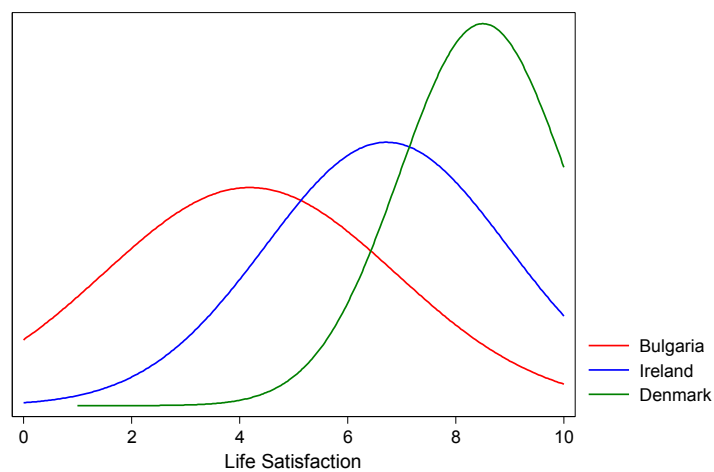
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## No variance at the household level



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(As usual) a mix of both



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## Useful utilities

We use `xtset` to tell Stata that our dataset is clustered (although this is not really necessary for most analyses)

We use `xtsum` to ask for descriptive statistics about our variables

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```

. xtset country
   panel variable:  country (balanced)

. xtsum stflife

```

Variable		Mean	Std. Dev.	Min	Max	Observations
stflife	overall	<b>6.783966</b>	<b>2.414616</b>	<b>0</b>	<b>10</b>	N = 5800
	between	<b>1.034484</b>	<b>4.185</b>	<b>8.5</b>		n = 29
	within	<b>2.190191</b>	<b>-1.351034</b>	<b>12.59897</b>		T = 200

NOTE: the grand mean (here 6.784) is added back to the within estimates to make the numbers comparable.

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## Intraclass Correlation ( $\rho$ )

$$\rho = \frac{\sigma_{country}^2}{\sigma_{country}^2 + \sigma_{individual}^2}$$

$$\rho = \frac{\psi}{\psi + \theta}$$

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## Estimating in Stata

We have a couple of options:

- `xtreg` is easy but can't do a lot
- `mixed` is more complicated and more flexible
  - called `xtmixed` in earlier versions of Stata
  - a few user-written postestimation commands work only after `xtmixed` (which still works)

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```
. xtreg stflife, i(country) mle
Iteration 0: log likelihood = -12845.508
Iteration 1: log likelihood = -12845.508

Random-effects ML regression      Number of obs   =    5,800
Group variable: country          Number of groups =     29

Random effects u_i ~ Gaussian    Obs per group:
                                   min =    200
                                   avg =   200.0
                                   max =    200

Log likelihood = -12845.508      Wald chi2(0)    =    0.00
                                   Prob > chi2     =      .
```

stflife	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
_cons	6.783966	.1887578	35.94	0.000	6.414007	7.153924
/sigma_u	1.004567	.1350565			.7718647	1.307424
/sigma_e	2.195497	.0204358			2.155807	2.235918
rho	.1731158	.0385865			.1079808	.2589746

```
LR test of sigma_u=0: chibar2(01) = 993.54      Prob >= chibar2 = 0.000
```

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```

. mixed stflife || country:
Performing EM optimization:
Performing gradient-based optimization:
Iteration 0:  log likelihood = -12845.508
Iteration 1:  log likelihood = -12845.508 (backed up)

Computing standard errors:
Mixed-effects ML regression              Number of obs   =    5,800
Group variable:  country                 Number of groups =     29

                                         obs per group:
                                         min =         200
                                         avg =        200.0
                                         max =         200

Log likelihood = -12845.508                wald chi2(0)    =      .
                                         Prob > chi2     =      .

```

stflife	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
_cons	6.783966	.1887578	35.94	0.000	6.414007 7.153924

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]
<b>country: Identity</b>			
var(_cons)	1.009155	.271346	.5957758 1.709356
var(Residual)	4.820209	.0897336	4.647504 4.999331

LR test vs. linear model:  $\text{chibar2}(01) = 993.54$       Prob >=  $\text{chibar2} = 0.0000$

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```

. estat icc
Intraclass correlation

```

Level	ICC	Std. Err.	[95% Conf. Interval]
country	.1731158	.0385865	.1098739 .2620427

We can say something like: “17% of the variance in life satisfaction is at the country level.”

This is why these are called **variance components models**.

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## Getting the individual estimates

```

predict u, reffects           // get the values
egen one = tag(country)      // pick one case per country
sort u
list country u if one, clean noobs

```

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country	u
BG	-2.49272
UA	-1.536227
HU	-1.047103
RU	-.9942981
AL	-.9289942
FR	-.8954188
LT	-.8819007
EE	-.6304559
XK	-.6163967
IT	-.5671842
PT	-.5326819
CZ	-.4402117
SK	-.390595
IE	-.1248178
PL	.2240138
ES	.3247775
SI	.4002492
CY	.411091
GB	.4114189
IL	.5766028
BE	.6491744
DE	.6868622
NL	.8796129
SE	1.007696
IS	1.014848
FI	1.180357
NO	1.316033
CH	1.348022
DK	1.648247

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