STATISTICAL HORIZONS

Multilevel and Mixed Models Using R

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Upcoming Seminar:

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By the end of tomorrow, you will…

- Get the "big picture" intuition of MLMs
- Understand what random intercepts and random slopes are when to use each one
- Know the difference between fixed and random effects and how to implement models that use both
- Know how to estimate MLMs (linear and logit) in R and interpret their results

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What are MLMs?

Most broadly, they are models that estimate parameters from data with:

- multiple observations from the same group (e.g., students in classes)
- repeated observations from the same person
  - sometimes called panel data or longitudinal data
  - not the primary focus here but *very similar*

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Adding α for the grand mean

$$y_{ij} = \alpha + \mu_j + \varepsilon_{ij}$$

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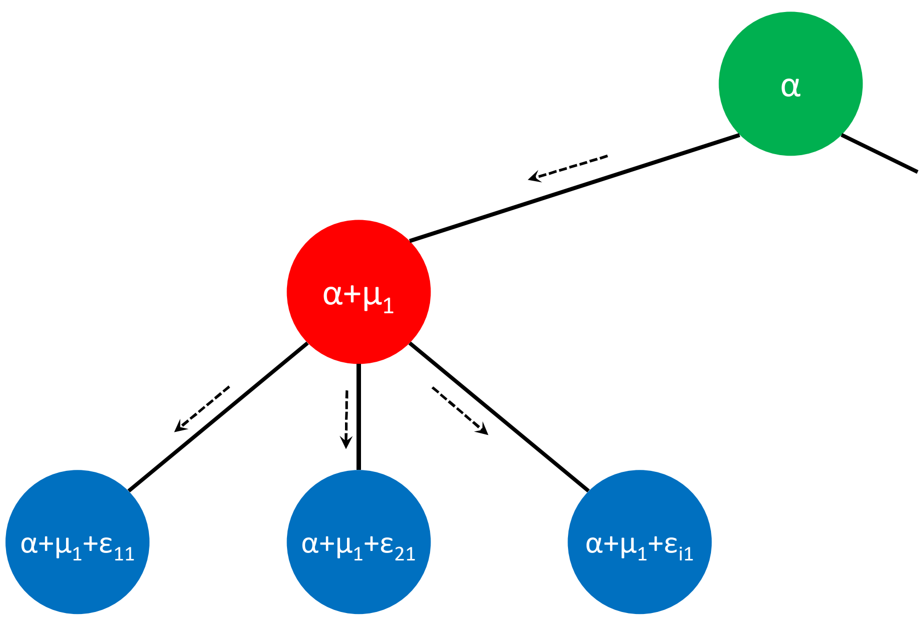

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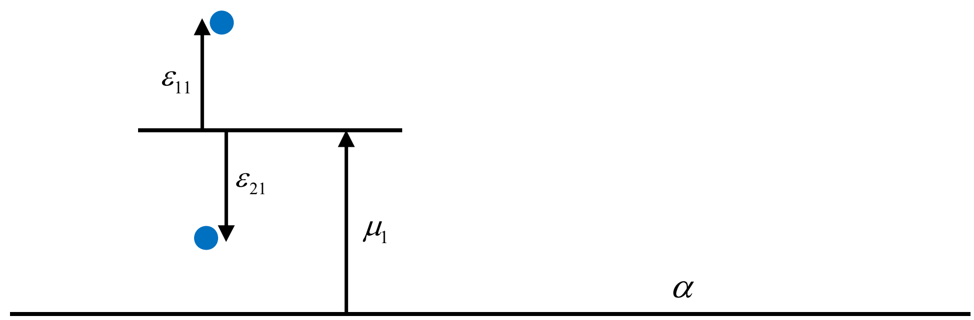

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Sometimes written in two equations
with distributional assumptions spelled out

$$\text{Level 1}: y_{ij} = \beta_{0j} + \varepsilon_{ij}, \varepsilon_{ij} \sim N(0, \sigma^2)$$
$$\text{Level 2}: \beta_{0j} = \beta_0 + \mu_j, \mu_j \sim N(0, \tau^2)$$

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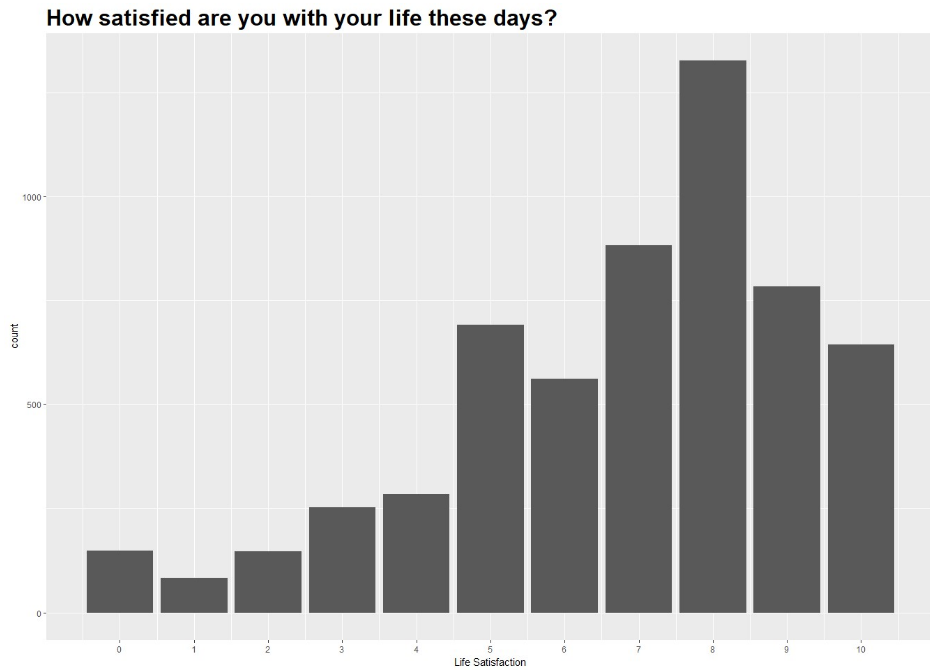

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```
> describe(ess_simple$stflife)
   vars    n mean   sd median trimmed  mad min max range  skew kurtosis   se
X1    1 5800 6.78 2.41      7    7.02 2.97   0  10    10 -0.86      0.3 0.03
> table(ess_simple$cntry)

 AL  BE  BG  CH  CY  CZ  DE  DK  EE  ES  FI  FR  GB  HU  IE  IL  IS  IT  LT  NL  NO  PL  PT  RU  SE  SI  SK  UA  XK 
200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 
```

Life Satisfaction

Mean	6.8
Variance	5.8

(remember these numbers)

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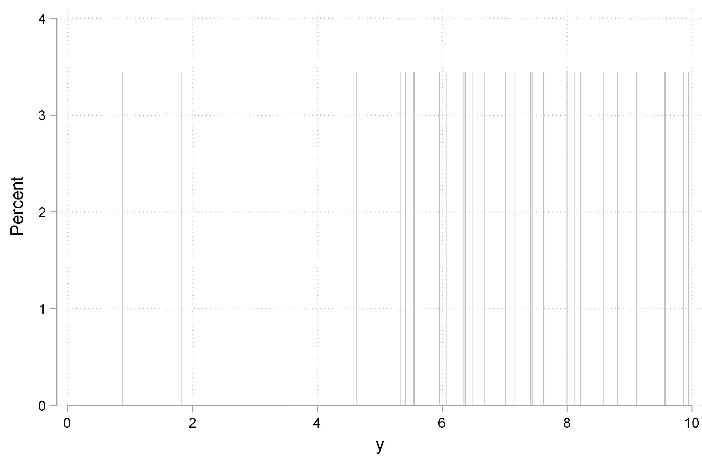
Two paths to the same variance: (1) all at the country level



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Two paths to the same variance: (2) all at the individual level

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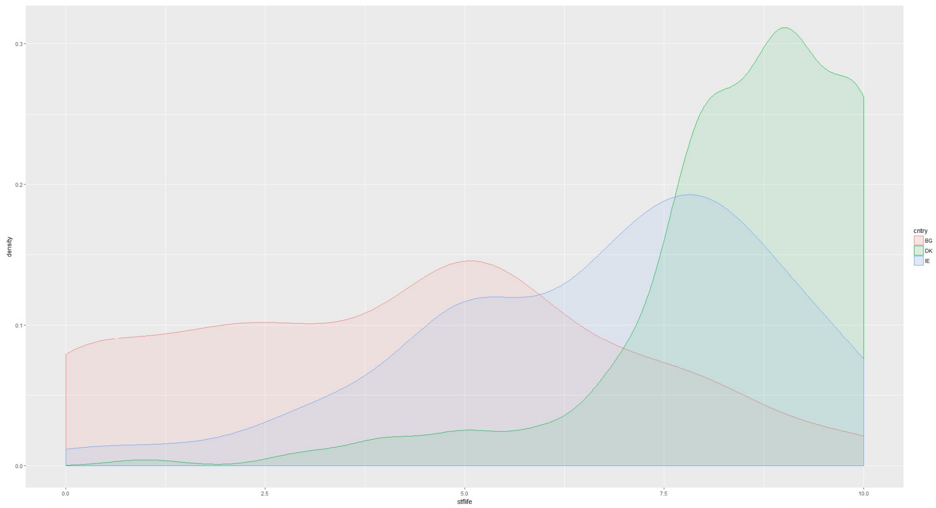
(As usual) a mix of both



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Intraclass Correlation (ρ)

$$\rho = \frac{\tau^2}{\tau^2 + \sigma^2}$$

$$ICC = \frac{\text{L2 variance}}{\text{total variance}}$$

$$\rho = \frac{\psi}{\psi + \theta}$$

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The simplest mixed model

$$y_{ij} = \beta_0 + \beta_1 x_{ij} + u_j + \varepsilon_{ij}$$

$$sat_{ij} = \beta_0 + \beta_1 ed_{ij} + u_j + \varepsilon_{ij}$$

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Basic `lme4` syntax

```
lmer(y ~ x + (1 | cluster), # cluster is L2 id
     data = df,             # df is data frame
     REML = FALSE)          # use maximum likelihood
```

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```
> # Basic mixed model
> m2 <- lmer(stflife ~ eduyrs + (1 | cntry),
+            data = ess_simple,
+            REML = FALSE)
> summary(m2)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: stflife ~ eduyrs + (1 | cntry)
   Data: ess_simple

     AIC      BIC   logLik deviance df.resid
 25661.9  25688.6 -12827.0  25653.9     5796

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.7077 -0.5339  0.1210  0.6780  2.6587

Random effects:
 Groups   Name        Variance Std.Dev.
 cntry    (Intercept) 0.9607   0.9801
 Residual             4.7905   2.1887
Number of obs: 5800, groups:  cntry, 29

Fixed effects:
            Estimate Std. Error t value
(Intercept)  6.20161    0.20752   29.88
eduyrs       0.04716    0.00773    6.10
```


	Null model	Null + education	
Fixed part			
Years of education		0.047	
Constant	6.784	6.202	
Random part			
L2 variance	1.009	0.961	
L1 variance	4.820	4.791	
ICC	.173	.167	This is now a *residual* ICC which isn't very useful

Adding random slopes: What *are* they?

Random slopes (a.k.a. random coefficients) are interactions between **observed** L1 variables and **unobserved** L2 variables.

They allow L1 slopes to differ between L2 clusters without accounting for **why** these differences might exist.

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Random coefficients/slopes

$$sat_{ij} = \underbrace{b_0 + u_{0j}}_{\text{intercept}} + (\underbrace{b_1 + u_{1j}}_{\text{slope}})ed_{ij} + \varepsilon_{ij}$$

A cluster-specific increment (or decrement) is added to (subtracted from) the "grand mean" (b_1) of the slope

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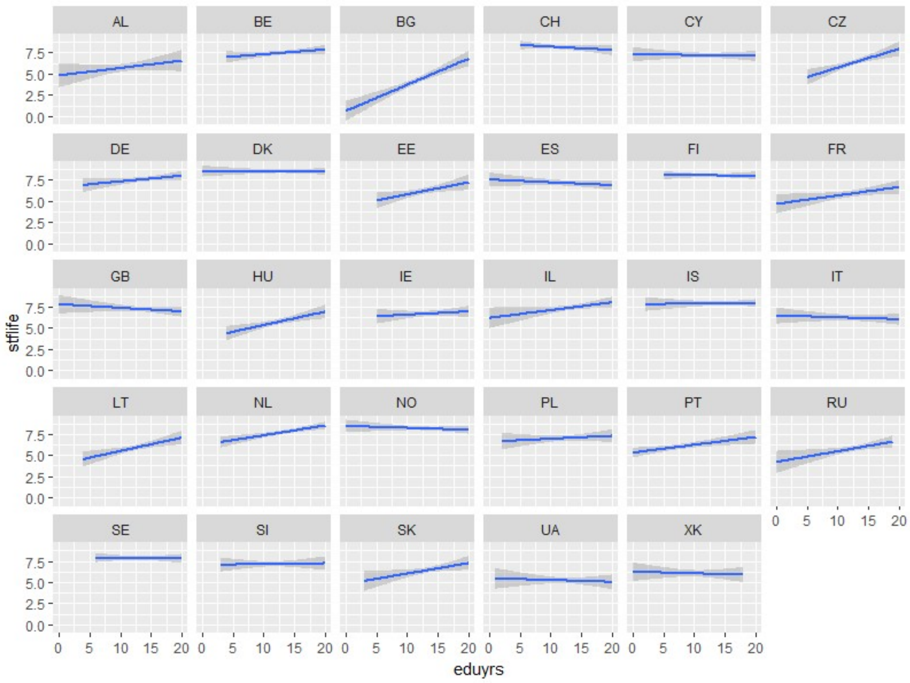

Looking at a graph like this might make us wonder if the effects differ systematically by country!

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Random coefficients/slopes syntax

```
lmer(y ~ x + (1 + x | cluster),   # x is now in both EQs
     data = df,
     REML = FALSE)
```

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The Between-Within (BW) method

For one within-cluster varying variable, *x*, (e.g., years of education) and one cluster-invariant variable, *z*, (e.g., GNI per capita), it would look like this:

$$y_{ij} = \beta_0 + \underbrace{\beta_1(x_{ij} - \overline{x}_j)}_{\text{within}} + \underbrace{\beta_2\overline{x}_j + \beta_3 z_j}_{\text{between}} + u_j + \varepsilon_{ij}$$

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BW model results

```
> summary(bw1)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: stflife ~ b.eduyrs + w.eduyrs + (1 | cntry)
   Data: ess_simple_bw

     AIC      BIC   logLik deviance df.resid
   25659    25692   -12824    25649     5795

Scaled residuals:
   Min      1Q Median     3Q    Max
-3.702 -0.530  0.118  0.677  2.657

Random effects:
 Groups   Name        Variance Std.Dev.
 cntry    (Intercept) 0.799    0.894
 Residual             4.790    2.189
Number of obs: 5800, groups:  cntry, 29

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.06758    1.74312    1.19
b.eduyrs     0.38192    0.14049    2.72
w.eduyrs     0.04631    0.00774    5.98
```

the w.eduyrs coefficient is *exactly* the same as any FE estimate

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	How satisfied with life as a whole	How satisfied with life as a whole
	B (CI)	*B (CI)*
Fixed Parts		
(Intercept)	6.784 (6.41 – 7.15) ***	2.068 (-1.35 – 5.48)
Education (between)		0.382 (0.11 – 0.66) **
Education (within)		0.046 (0.03 – 0.06) ***
Random Parts		
σ^2	4.820	4.790
$\tau_{00,\ cntry}$	1.009	0.799
N_{cntry}	29	29
ICC_{cntry}	0.173	0.143
Observations	5800	5800
Notes		* *p*<.05 ** *p*<.01 *** *p*<.001

$$R^2_{L1} = \frac{4.820 - 4.790}{4.820} = .006$$

$$R^2_{L2} = \frac{1.009 - .799}{1.009} = .208$$

$$R^2_{overall} = \frac{(4.820 + 1.009) - (4.790 + .799)}{(4.820 + 1.009)} = .041$$

Mean education accounts for 21% of the between-country variance in life satisfaction. Within-country education differences don't account for much within-country variance.

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Adding L2-only variables

```
> bw2 <- lmer(stflife ~ b.eduyrs + w.eduyrs + gnipc1000 + (1 | cntry),
+             REML = FALSE,
+             data = ess_simple_bw)
> summary(bw2)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: stflife ~ b.eduyrs + w.eduyrs + gnipc1000 + (1 | cntry)
   Data: ess_simple_bw

     AIC      BIC   logLik deviance df.resid
   25638    25678   -12813    25626     5794

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.726 -0.525  0.120  0.674  2.639

Random effects:
 Groups   Name        Variance Std.Dev.
 cntry    (Intercept) 0.357    0.597
 Residual             4.790    2.189
Number of obs: 5800, groups:  cntry, 29

Fixed effects:
            Estimate Std. Error t value
(Intercept) 3.58579   1.21396    2.95
b.eduyrs    0.18010   0.10167    1.77
w.eduyrs    0.04631   0.00774    5.98
gnipc1000   0.03111   0.00536    5.81
```

You don't have to do anything special to include L2 variables. Just add them into the fixed portion like any other variable.

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Comparing many models

Many decisions to make, for example:

- Do I need to split out education into between/within?
- Do I need to include a specific variable at all?
- Do I need random slopes on the L1 variables?

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AIC/BIC of all models (so far)

Model	Specification	AIC	BIC
M1	random intercepts	25,697	25,717
M2	RI + eduyrs	25,662	25,689
M3	RI + eduyrs*	25,610	25,650
BW1	RI + b/w.eduyrs	25,659	25,692
BW2	RI + b/w.eduyrs + GDP	25,638	25,678
BW3	RI + b/w*.eduyrs + GDP	**25,592**	25,645
M4	RI + eduyrs* + GDP	**25,592**	**25,638**

* signifies random slopes

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Cross-level interactions

We might want to know whether the *context* of a cluster (i.e., cluster-level attributes) conditions the effects of a within-cluster difference.

For example, do within-country educational differences matter *less* for determining life satisfaction in richer countries?

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Centering and interactions

We center variables to make their related coefficients more interpretable.

If done properly, centering has no effect on **model fit** but facilitates **model interpretation**.

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Limited Dependent Variables

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What can we do with `lme4`?

```
binomial(link = "logit")
gaussian(link = "identity")
Gamma(link = "inverse")
inverse.gaussian(link = "1/mu^2")
poisson(link = "log")
quasi(link = "identity", variance = "constant")
quasibinomial(link = "logit")
quasipoisson(link = "log")
```

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