STATISTICAL HORIZONS

Linear Regression

Paul D. Allison, Ph.D.

Upcoming Seminar:
September 13-October 11, 2021, On Demand

Linear Regression Analysis

Paul D. Allison, Ph.D.

What is Linear Regression?

A statistical technique for studying the relationship between a single dependent variable *y* and one or more independent variables (the *x*'s).

Goal: A linear equation of the form

$$\hat{y} = b_0 + b_1 x_1 + \ldots + b_k x_k$$

where *y*-hat is the predicted value of *y* and the *b*'s are the estimated "regression coefficients".

Two main uses:

1. Generate predictions about *y* based on knowledge of *x*'s.
2. Estimate and test hypotheses about the "causal" effect of each *x* on *y*, "controlling" for the other *x*'s.

Prediction

How does Zillow produce an estimate of a home's value?

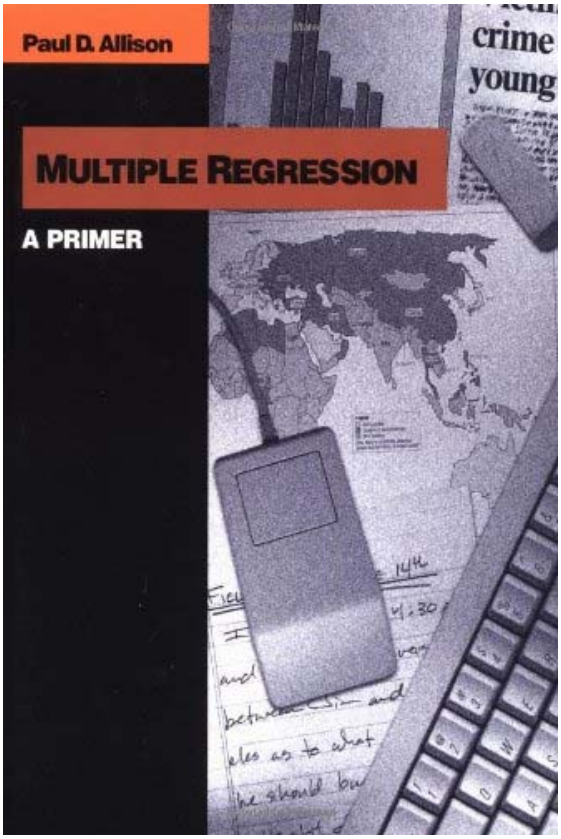
- Collect data on sale price of millions of homes.
- Also collect data on "location, lot size, square footage, number of bedrooms and bathrooms, actual property taxes paid, exceptions to tax, actual sale prices over time of the home itself and comparable recent sales of nearby homes."
- Formula is "proprietary" but I'm pretty sure it's based on some form of linear regression.
- Potential complications: Different information is available for different homes, how to deal with spatial information, change over time, different real estate markets, etc.

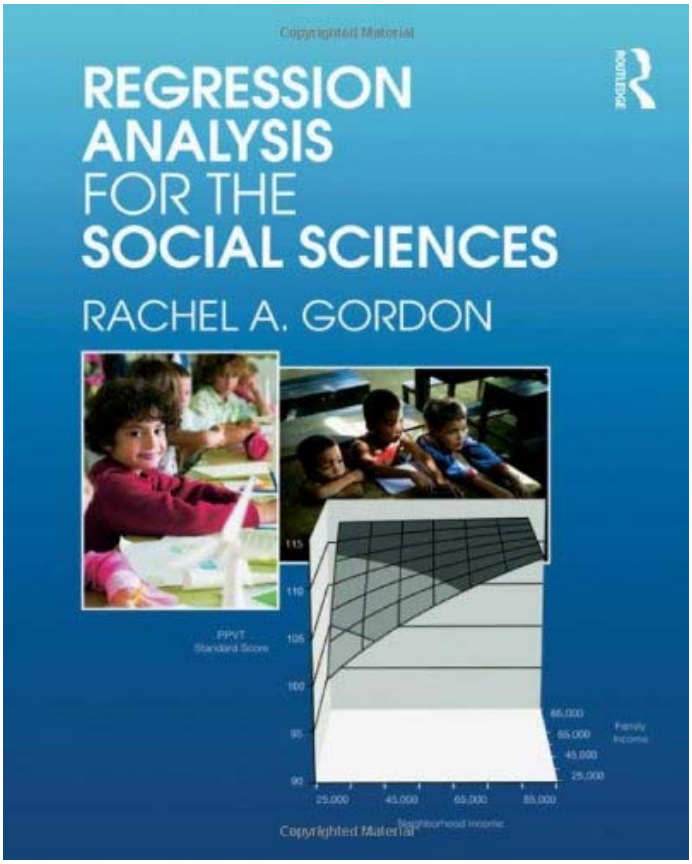
Causal Effects

Does divorce lead to bad outcomes for children?

Allison, Paul D. and Frank F. Furstenberg, Jr. (1989) "How marital dissolution affects children: Variations by age and sex." *Developmental Psychology* 25: 540-549.

- For 1197 children, we compared those from intact families with those whose parents divorced or separated (328).
- Children of divorce did worse on measures of delinquency, hyperactivity, academic difficulty, distress, etc.
- Problem: Couples who divorced differed in many other respects from those who did not.
- Solution: Estimate linear regressions that controlled for child's age, race, sex, birth order, region of residence, mother's education, religious preference, age at birth of the child, age at birth of first child, and foreign or U.S. birth.
- Result: Most "effects" of divorce remained strong after controls.


Paul D. Allison
MULTIPLE REGRESSION
A PRIMER
crime
young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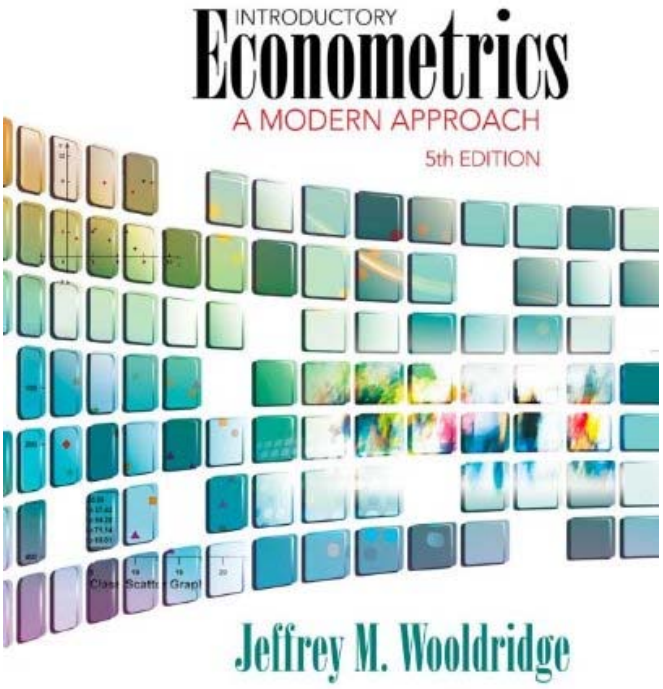


Beauty Data Set

Source: Hamermesh, D.S. and J.E. Biddle (1994) "Beauty and the Labor Market." *American Economic Review* 84: 1174-1194.

1260 employed adults in 1977 in the US

wage	**hourly wage (THIS WILL BE OUR DEPENDENT VARIABLE)**
exper	**years of workforce experience**
looks	**from 1 to 5 (as rated by interviewers)**
union	**=1 if union member , else 0**
goodhlth	**=1 if good health, else 0**
black	**=1 if black, else 0**
female	**=1 if female , else 0**
married	**=1 if married, else 0**
south	**=1 if live in south, else 0**
bigcity	**=1 if live in big city, else 0**
smllcity	**=1 if live in small city, else 0**
service	**=1 if service industry, else 0**
educ	**years of schooling**

Questions

Can we construct a “good” model to predict hourly wages?

Does more education lead to higher wages?

- Is that true even if we control for other characteristics?
- What’s the magnitude of the effect?

Do women make less than men?

Do blacks make less than non-blacks?

Do more attractive people get higher wages?

Do union workers make more than non-union workers?

Preliminary Steps

Get familiar with the data

- Check the distribution of each variable.
- Compute means, minima, maxima, frequency tables, histograms, etc.
- Look for outliers and coding errors.
- Check for missing data.

Using Stata

```
. use "C:\data\beauty.dta"
. summarize
```

Variable	Obs	Mean	Std. Dev.	Min	Max
wage	1260	6.30669	4.660639	1.02	77.72
lwage	1260	1.6588	.5945075	.0198026	4.353113
exper	1260	18.20635	11.96349	0	48
looks	1260	3.185714	.6848774	1	5
union	1260	.2722222	.4452804	0	1
goodhlth	1260	.9333333	.2495429	0	1
black	1260	.0738095	.2615645	0	1
female	1260	.3460317	.4758923	0	1
married	1260	.6912698	.462153	0	1
south	1260	.1746032	.3797781	0	1
bigcity	1260	.2190476	.4137652	0	1
smllcity	1260	.4666667	.4990857	0	1
service	1260	.2738095	.4460895	0	1
educ	1260	12.56349	2.624489	5	17

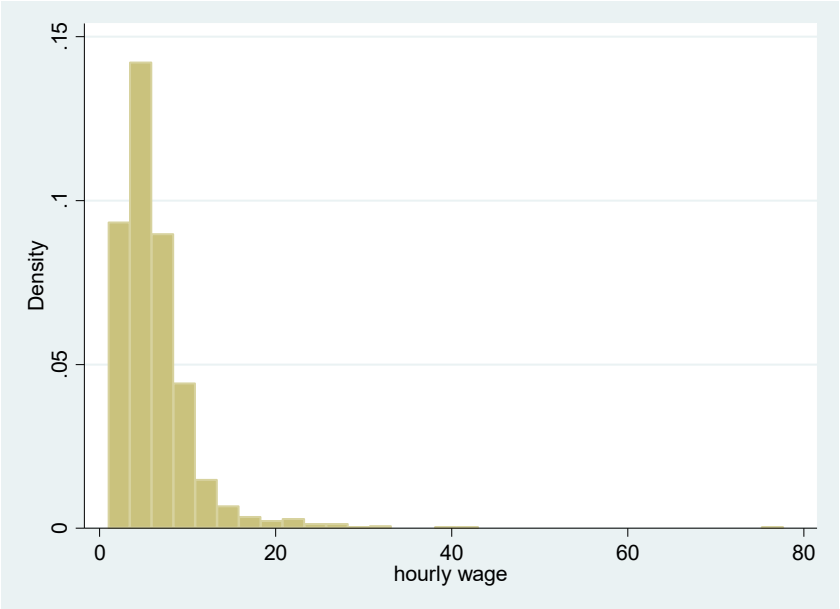
Using SAS

```
PROC MEANS DATA=my.beauty; RUN;
```

Variable	N	Mean	Std Dev	Minimum	Maximum
wage	1260	6.3066905	4.6606390	1.0200000	77.7200000
lwage	1260	1.6587996	0.5945075	0.0198026	4.3531130
exper	1260	18.2063492	11.9634853	0	48.0000000
looks	1260	3.1857143	0.6848774	1.0000000	5.0000000
union	1260	0.2722222	0.4452804	0	1.0000000
goodhlth	1260	0.9333333	0.2495429	0	1.0000000
black	1260	0.0738095	0.2615645	0	1.0000000
female	1260	0.3460317	0.4758923	0	1.0000000
married	1260	0.6912698	0.4621530	0	1.0000000
south	1260	0.1746032	0.3797781	0	1.0000000
bigcity	1260	0.2190476	0.4137652	0	1.0000000
smllcity	1260	0.4666667	0.4990857	0	1.0000000
service	1260	0.2738095	0.4460895	0	1.0000000
educ	1260	12.5634921	2.6244892	5.0000000	17.0000000

Histogram of wage

In Stata: **`hist wage`**



Distribution of looks

```
. tab looks

from 1 to 5 |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |         13        1.03        1.03
          2 |        142       11.27       12.30
          3 |        722       57.30       69.60
          4 |        364       28.89       98.49
          5 |         19        1.51      100.00
------------+-----------------------------------
      Total |      1,260      100.00
```

Note:
1=homely, 2=quite plain, 3=average, 4=good looking, 5=strikingly beautiful

Stata Regression Predicting Wages

```
regress wage exper looks union goodhlth black female married
   south bigcity smllcity service educ
```

```
      Source |       SS       df       MS              Number of obs =    1260
-------------+------------------------------           F( 12,  1247) =   29.28
       Model |  6011.43607    12  500.953006           Prob > F      =  0.0000
    Residual |  21336.0031  1247  17.1098662           R-squared     =  0.2198
-------------+------------------------------           Adj R-squared =  0.2123
       Total |  27347.4392  1259  21.7215561           Root MSE      =  4.1364

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       exper |    .078945   .0106672     7.40   0.000     .0580174    .0998725
       looks |   .4138739   .1742518     2.38   0.018     .0720148     .755733
       union |   .6114653   .2670238     2.29   0.022     .0875999    1.135331
    goodhlth |  -.0524494   .4755118    -0.11   0.912    -.9853408    .8804419
       black |  -.1106673   .4612814    -0.24   0.810    -1.015641    .7943061
      female |  -2.127858   .2763716    -7.70   0.000    -2.670063   -1.585654
     married |   .8213589   .2744173     2.99   0.003     .2829883    1.359729
       south |   .3572703   .3116461     1.15   0.252    -.2541384    .9686789
     bigcity |   1.720111   .3363336     5.11   0.000     1.060269    2.379954
    smllcity |   .5875548   .2736183     2.15   0.032     .0507518    1.124358
     service |   -.478788   .2882098    -1.66   0.097    -1.044218    .0866417
        educ |   .4246026   .0500969     8.48   0.000     .3263191    .5228861
       _cons |  -2.306654   .9797571    -2.35   0.019    -4.228808   -.3844996
```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       _cons |  -2.306654   .9797571    -2.35   0.019    -4.228808   -.3844996
```

This is an F test of the null hypothesis that all 12 coefficients are 0. The p-value is extremely low, so we can reject that hypothesis and conclude that at least one coefficient is not 0.