

# Longitudinal Data Analysis Using Stata

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## Panel Data

Data in which variables are measured at multiple points in time for the same individuals.

Response variable  $y_{it}$  with  $t = 1, 2, \dots, T$

Vector of predictor variables  $x_{it}$ .

Some of these may vary with time, others may not.

Assume, for now, that time points are the same for everyone in the sample. (For some methods that assumption is not essential).

## Why Are Panel Data Desirable?

In *Econometric Analysis of Panel Data* (2008), Baltagi lists six potential benefits of panel data:

1. Ability to control for individual heterogeneity.
2. More informative data: more variability, less collinearity, more degrees of freedom and more efficiency.
3. Better ability to study the dynamics of adjustment. For example, a cross-sectional survey can tell you what proportion of people are unemployed, but a panel study can tell you the distribution of spells of unemployment.
4. Ability to identify and measure effects that are not detectable in pure cross-sections or pure time series. For example, if you want to know whether union membership increases or decreases wages, you can best answer this by observing what happens when workers move from union to non-union jobs, and vice versa.
5. Ability to construct and test more complicated behavioral models than with purely cross-section or time-series data. For example, distributed lag models may require fewer restrictions with panel data than with pure time-series data.
6. Avoidance of aggregation bias. A consequence of the fact that most panel data are micro-level data.

## My List

1. Ability to control for unobservables.

Accomplished by fixed effects methods.

2. Ability to resolve causal ordering: Does  $y$  cause  $x$  or does  $x$  cause  $y$ ?

Accomplished by simultaneous estimation of models with lagged predictors.

Methods for doing this are still relatively undeveloped and underutilized.

3. Ability to study the effect of a “treatment” on the trajectory of an outcome (or, equivalently, the change in a treatment effect over time).

## Problems with Panel Data

1. Attrition and missing data.

2. Statistical dependence among multiple observations from the same individual.

- Repeated observations on the same individual are likely to be positively correlated. Individuals tend to be persistently high or persistently low.
- But conventional statistical methods assume that observations are independent.
- Consequently, estimated standard errors tend to be too low, leading to test statistics that are too high and p-values that are too low.
- Also, conventional parameter estimates may be statistically inefficient (true standard errors are higher than necessary).
- Many different methods to correct for dependence:

- Robust standard errors
  - Generalized estimating equations (GEE)
  - Random effects (mixed) models
  - Fixed-effects models
- Many of these methods can also be used for clustered data that are not longitudinal, e.g., students within classrooms, people within neighborhoods.

## Software

I'll be using Stata 14, with a focus on the **xt** and **me** commands.

These commands require that the data be organized in the “long form” so that there is one record for each individual at each time point, with an ID number that is the same for all records for the same individual, and a variable that indicates which time point the record comes from.

All of the methods described here can also be implemented in SAS.

# Linear Models for Quantitative Response

Notation:

$y_{it}$  is the value of the response variable for individual  $i$  at time  $t$ .

$z_i$  is a column vector of variables that describe individuals but do not vary over time

$x_{it}$  is a column vector of variables that vary both over individuals and over time

Basic model:

$$y_{it} = \mu_t + \beta x_{it} + \gamma z_i + \varepsilon_{it}, \quad i=1, \dots, n ; t=1, \dots, T$$

where  $\varepsilon$  is a random error term with mean 0 and constant variance, assumed to be uncorrelated with  $x$  and  $z$ .  $\beta$  and  $\gamma$  are row vectors of coefficients.

No lags, different intercepts at each time point, coefficients the same at all time points.

Consider OLS (ordinary least squares) estimation.

- Coefficients will be unbiased but not efficient.
- Estimated standard errors will be too low because  $\text{corr}(\varepsilon_{it}, \varepsilon_{it'}) \neq 0$

## Example:

581 children interviewed in 1990, 1992, and 1994 as part of the National Longitudinal Survey of Youth (NLSY).

Time-varying variables:

ANTI antisocial behavior, measured with a scale ranging from 0 to 6.

SELF self-esteem, measured with a scale ranging from 6 to 24.

POV poverty status of family, coded 1 for in poverty, otherwise 0.

Time-invariant variables:

BLACK 1 if child is black, otherwise 0

HISPANIC 1 if child is Hispanic, otherwise 0

CHILDAGE child's age in 1990

MARRIED 1 if mother was currently married in 1990, otherwise 0

GENDER 1 if female, 0 if male

MOMAGE mother's age at birth of child

MOMWORK 1 if mother was employed in 1990, otherwise 0

Original data set `nlsy.dta` has 581 records, one for each child, with different names for the variables at each time point, e.g., ANTI90, ANTI92 and ANTI94.

We can convert the data into a set of 1743 records, one for each child in each year using the `reshape` command:

```
use c:\data\nlsy.dta, clear  
gen id = _n  
reshape long anti self pov, i(id) j(year)
```

## save persyr3, replace

Data	wide	->	long
Number of obs.	581	->	1743
Number of variables	17	->	12
j variable (3 values)		->	year
xij variables:			
	anti90 anti92 anti94	->	anti
	self90 self92 self94	->	self
	pov90 pov92 pov94	->	pov

Note:

The time-invariant variables are repeated across the multiple records for each child.

The variable **id** has a unique ID number for each child.

The variable **year** has values of 90, 92 or 94.

Now we'll do OLS regression, with no correction for dependence

```
reg anti self pov black hispanic chldage married  
gender momage momwork i.year
```



Source	SS	df	MS	Number of obs = 1743		
Model	380.85789	11	34.6234446	F( 11, 1731) = 15.16		
Residual	3952.25743	1731	2.28322208	Prob > F = 0.0000		
-----				R-squared = 0.0879		
Total	4333.11532	1742	2.48743704	Adj R-squared = 0.0821		
-----				Root MSE = 1.511		
anti	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
self	-.0741425	.0109632	-6.76	0.000	-.095645	-.0526401
pov	.4354025	.0855275	5.09	0.000	.2676544	.6031505
black	.1678622	.0881839	1.90	0.057	-.0050959	.3408204
hispanic	-.2483772	.0948717	-2.62	0.009	-.4344523	-.0623021
childage	.087056	.0622121	1.40	0.162	-.0349628	.2090747
married	-.0888875	.087227	-1.02	0.308	-.2599689	.082194
gender	-.4950259	.0728886	-6.79	0.000	-.637985	-.3520668
momage	-.0166933	.0173463	-0.96	0.336	-.0507153	.0173287
momwork	.2120961	.0800071	2.65	0.008	.0551754	.3690168
year						
92	.0521538	.0887138	0.59	0.557	-.1218437	.2261512
94	.2255775	.0888639	2.54	0.011	.0512856	.3998694
_cons	2.675312	.7689554	3.48	0.001	1.167132	4.183491

## Problems:

Although the coefficients are unbiased, they are not “efficient.” An estimator is said to be efficient if it has minimal sampling variability. The true standard errors are optimally small.

More important, estimated standard errors and  $p$ -values are probably too low

## Solution 1: Robust standard errors

Also known as Huber-White standard errors, sandwich estimates, or empirical standard errors.

For OLS linear models, conventional standard errors are obtained by first calculating the estimated covariance matrix of the coefficient estimates:

$$s^2(\mathbf{X}'\mathbf{X})^{-1}$$

where  $\mathbf{X}$  is a matrix of dimension  $Tn \times K$  (the number of coefficients) and  $s^2$  is the residual variance. Standard errors are obtained by taking the square roots of the main diagonal elements of this matrix.

The formula for the robust covariance estimator is

$$\hat{\mathbf{V}} = (\mathbf{X}'\mathbf{X})^{-1} \left( \sum_i \mathbf{X}'_i \hat{\mathbf{u}}_i \hat{\mathbf{u}}'_i \mathbf{X}_i \right) (\mathbf{X}'\mathbf{X})^{-1}$$

where  $\mathbf{X}_i$  is a  $T \times K$  matrix of covariate values for individual  $i$  and

$$\hat{\mathbf{u}}_i = \mathbf{y}_i - \mathbf{X}_i \hat{\boldsymbol{\beta}}$$

is a  $T \times 1$  vector of residuals for individual  $i$ . The robust standard errors are the square roots of the main diagonal elements of  $\hat{\mathbf{V}}$ .

In Stata, this method can be implemented with most regression commands using the **vce** option:

```
reg anti self pov black hispanic chldage married  
momage gender momwork i.year, vce(cluster id)
```

<b>Linear regression</b>	<b>Number of obs = 1743</b>
	<b>F( 11, 580) = 8.99</b>
	<b>Prob &gt; F = 0.0000</b>
	<b>R-squared = 0.0879</b>
	<b>Root MSE = 1.511</b>
<b>(Std. Err. adjusted for 581 clusters in id)</b>	

anti	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
self	-.0741425	.0133707	-5.55	0.000	-.1004034	-.0478816
pov	.4354025	.1093637	3.98	0.000	.2206054	.6501995
black	.1678622	.1309221	1.28	0.200	-.0892769	.4250014
hispanic	-.2483772	.1341785	-1.85	0.065	-.5119122	.0151578
childage	.087056	.0939055	0.93	0.354	-.0973804	.2714923
married	-.0888875	.1336839	-0.66	0.506	-.3514509	.173676
momage	-.0166933	.0241047	-0.69	0.489	-.0640364	.0306498
gender	-.4950259	.1057334	-4.68	0.000	-.7026929	-.2873589
momwork	.2120961	.1189761	1.78	0.075	-.0215803	.4457725
year						
92	.0521538	.0540096	0.97	0.335	-.0539244	.158232
94	.2255775	.0641766	3.51	0.000	.0995306	.3516245
_cons	2.675312	1.138426	2.35	0.019	.4393717	4.911252

Although coefficients are the same, almost all the standard errors are larger. This makes a crucial difference for MOMWORK, BLACK and HISPANIC.

Notes:

- It's possible for robust standard errors to be *smaller* than conventional standard errors.
- You generally see a bigger increase in the standard errors for time-invariant variables than for time-varying variables.
- Robust SEs are also robust to heteroskedasticity.
- For small samples, robust standard errors may be inaccurate and have low power. To get reasonably accurate results, you need *at least 20* clusters if they are approximately balanced, 50 if they are unbalanced.

## Solution 2: Generalized Estimating Equations (GEE, population averaged models)

For linear models, this is equivalent to feasible generalized least squares (GLS).

The attraction of this method is that it produces efficient estimates of the coefficients (i.e., true standard errors will be optimally small). It does this by taking the over-time correlations into account when producing the estimates.

Conventional least squares estimates are given by the matrix formula

$$(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

GLS estimates are obtained by

$$(\mathbf{X}'\hat{\mathbf{\Omega}}^{-1}\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{\Omega}}^{-1}\mathbf{y}$$

where  $\hat{\mathbf{\Omega}}$  is an estimate of the covariance matrix for the error terms. For panel data, this will typically be a “block-diagonal” matrix. For example, if the sample consists of three people with two observations each, the covariance matrix will look like

$$\hat{\mathbf{\Omega}} = \begin{bmatrix} \hat{\sigma}_{11} & \hat{\sigma}_{12} & 0 & 0 & 0 & 0 \\ \hat{\sigma}_{12} & \hat{\sigma}_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{\sigma}_{11} & \hat{\sigma}_{12} & 0 & 0 \\ 0 & 0 & \hat{\sigma}_{12} & \hat{\sigma}_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & \hat{\sigma}_{11} & \hat{\sigma}_{12} \\ 0 & 0 & 0 & 0 & \hat{\sigma}_{12} & \hat{\sigma}_{22} \end{bmatrix}$$

In Stata, the method can be implemented with the **xtgee** command. It's convenient to first declare the data set to be a time-series cross-section data set using the **xtset** command.

**xtset id year**

**panel variable: id (strongly balanced)**  
**time variable: year, 90 to 94, but with gaps**  
**delta: 1 unit**

**xtgee anti self pov black hispanic childage married  
gender momage momwork i.year**

GEE population-averaged model		Number of obs	=	1743
Group variable:	id	Number of groups	=	581
Link:	identity	Obs per group: min	=	3
Family:	Gaussian	avg	=	3.0
Correlation:	exchangeable	max	=	3
		Wald chi2(11)	=	105.37
Scale parameter:	2.275542	Prob > chi2	=	0.0000

  

anti	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
self	-.0620764	.0094874	-6.54	0.000	-.0806715 -.0434814
pov	.2471376	.080136	3.08	0.002	.090074 .4042013
black	.2267537	.1249995	1.81	0.070	-.018241 .4717483
hispanic	-.2182088	.137456	-1.59	0.112	-.4876177 .0512001
childage	.0884559	.0905831	0.98	0.329	-.0890836 .2659955
married	-.0495647	.1257172	-0.39	0.693	-.295966 .1968365
gender	-.4834488	.1059245	-4.56	0.000	-.6910571 -.2758405
momage	-.0219197	.0251467	-0.87	0.383	-.0712064 .0273669
momwork	.2611318	.1140581	2.29	0.022	.037582 .4846815
year					
92	.0473396	.0585299	0.81	0.419	-.0673769 .162056
94	.2163811	.0587023	3.69	0.000	.1013267 .3314355
_cons	2.531431	1.089759	2.32	0.020	.3955422 4.667321

By default, the standard errors are “model based”. Although corrected for dependence, they are sensitive to the particular correlation structure that is specified.

The default correlation structure is “exchangeable”, which means that the correlations between the dependent variables at different points in time are all the same. To see the estimated correlations, use the command:

**estat wcorr**

Estimated within-id correlation matrix R:

	c1	c2	c3
r1	1		
r2	.5636779	1	
r3	.5636779	.5636779	1

To get robust standard errors (that aren't sensitive to the correlation structure), simply add the robust option to the `xtgee` command:

```
xtgee anti self pov black hispanic childage married  
momage gender momwork i.year, vce(robust)
```

```
GEE population-averaged model      Number of obs      =      1743
Group variable:                    id      Number of groups   =      581
Link:                               identity  Obs per group: min =      3
Family:                             Gaussian  avg               =      3.0
Correlation:                        exchangeable  max               =      3
                                     Wald chi2(11)     =      90.65
Scale parameter:                    2.275542        Prob > chi2       =      0.0000
```

(Std. Err. adjusted for clustering on id)

anti	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
self	-.0620764	.0101609	-6.11	0.000	-.0819915 -.0421614
pov	.2471376	.0835503	2.96	0.003	.0833821 .4108932
black	.2267537	.130129	1.74	0.081	-.0282945 .4818019
hispanic	-.2182088	.1337172	-1.63	0.103	-.4802896 .043872
childage	.0884559	.0939841	0.94	0.347	-.0957496 .2726615
married	-.0495647	.1341853	-0.37	0.712	-.3125631 .2134336
momage	-.0219197	.0239744	-0.91	0.361	-.0689087 .0250693
gender	-.4834488	.1058324	-4.57	0.000	-.6908764 -.2760212
momwork	.2611318	.1163266	2.24	0.025	.0331359 .4891276
year					
92	.0473396	.0535429	0.88	0.377	-.0576025 .1522817
94	.2163811	.0634953	3.41	0.001	.0919327 .3408295
_cons	2.531431	1.128098	2.24	0.025	.3203999 4.742463

With only three time points, you're probably better off specifying an "unstructured" model that imposes no pattern on the correlation matrix:

```
xtgee anti self pov black hispanic childage married  
momage gender momwork i.year, vce(r) corr(uns)
```

GEE population-averaged model		Number of obs	=	1743
Group and time vars:		id year	Number of groups	= 581
Link:	identity	Obs per group: min	=	3
Family:	Gaussian	avg	=	3.0
Correlation:	unstructured	max	=	3
		Wald chi2(11)	=	94.51
Scale parameter:	2.273983	Prob > chi2	=	0.0000

(Std. Err. adjusted for clustering on id)

---

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
anti						
self	-.0629882	.0101177	-6.23	0.000	-.0828186	-.0431579
pov	.268169	.0834573	3.21	0.001	.1045958	.4317423
black	.2129144	.1298973	1.64	0.101	-.0416796	.4675084
hispanic	-.2281683	.1329107	-1.72	0.086	-.4886684	.0323318
childage	.0852542	.0934659	0.91	0.362	-.0979356	.2684441
married	-.050604	.1335751	-0.38	0.705	-.3124065	.2111984
momage	-.0202607	.02389	-0.85	0.396	-.0670842	.0265628
gender	-.4860039	.1054709	-4.61	0.000	-.692723	-.2792847
momwork	.2525486	.1160187	2.18	0.029	.0251561	.479941
year						
92	.0477502	.0535456	0.89	0.373	-.0571972	.1526976
94	.2171697	.0635099	3.42	0.001	.0926927	.3416468
_cons	2.548914	1.121399	2.27	0.023	.3510115	4.746816

```
estat wcorr
```

Estimated within-id correlation matrix R:

	c1	c2	c3
r1	1		
r2	.5512489	1	
r3	.5193459	.6186195	1

With many time points the number of unique correlations will get large:  $T(T-1)/2$ . And unless the sample is also large, estimates of all these parameters may be unreliable.

In that case, consider restricted models:

TYPE	Description	Formula
AR#	Autoregressive of order #	$\varepsilon_{it} = \sum_{j=1}^{\#} \theta_j \varepsilon_{it-j} + v_{it}$
STA#	Stationary of order #	$\rho_{ts} = \rho_{ t-s }$ when $ t-s  \leq \#$ , otherwise $\rho_{ts} = 0$
NON#	Non-stationary of order #	$\rho_{ts} = \rho_{ts}$ when $ t-s  \leq \#$ , otherwise $\rho_{ts} = 0$

Results will often be robust to choice of correlation structure, but sometimes it can make a big difference. An autoregressive structure of order 1 is usually too restrictive: the correlation goes down too rapidly with the time distance.

GEE can handle missing data on the response variable (or unbalanced panels) under the assumption that the data are missing completely at random, or that missingness depends only on the predictors. It does not allow missingness on  $y$  at one time to depend on observed values of  $y$  at other times.