STATISTICAL HORIZONS

Difference in Differences

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Upcoming Seminar:

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Lecture 1: Introduction to Difference-in-Differences

Potential outcomes and the canonical 2x2 DiD setup

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Introduction and DiD Popularity

Importance of Empirical Research

- The availability of richer datasets have changed Social Sciences during the last 40 years.
- Currie, Kleven and Zwiers (2020) show that the fraction of empirical research keeps rising.
- A very common goal of empirical research is to uncover/highlight the **casual effect** of a given policy intervention.

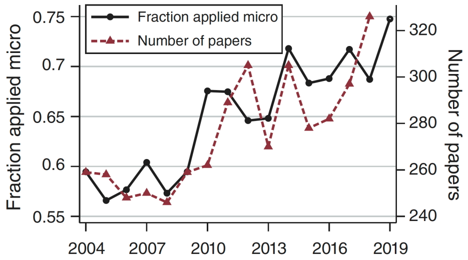

FIGURE 1. APPLIED MICROECONOMICS ARTICLES IN TOP-FIVE JOURNALS

Note: This figure shows the fraction of papers in top-five journals that report an applied microeconomics JEL code (left axis) and the total number of papers in the top-five journals (right axis).

The boom of Experimental and Quasi-Experimental Methods

- Back in the 80's, Leamer (1983) wrote the very influencial paper, "Let's Take the Con Out of Econometrics".

> "Hardly anyone takes data analysis seriously. Or perhaps more accurately, hardly anyone takes anyone data analysis seriously."

- Since then, we have witnessed a "credibility revolution" in social sciences (Angrist and Pischke, 2010).
- Focus on the quality of empirical **"research designs"**.
- Importance of being **explicit and transparent** about the type of variation in the data you are leveraging to recover causal effects.

The boom of Experimental and Quasi-Experimental Methods

Currie et al. (2020) documented this change well

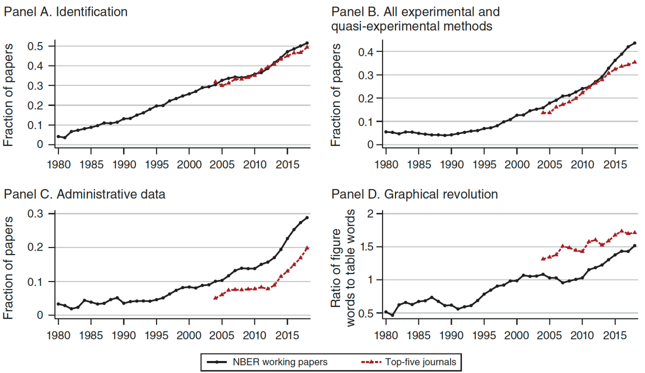

FIGURE 2. THE CREDIBILITY REVOLUTION

Notes: This figure shows different dimensions of the "credibility revolution" in economics: identification (panel A), all experimental and quasi-experimental methods (panel B), administrative data (panel C), and the graphical revolution (panel D). Panel D shows the ratio of the number of "figure" terms to the number of "table" terms mentioned. See Table A.I for a list of terms. The series show five-year moving averages.

The boom of Experimental and Quasi-Experimental Methods

5 main "research designs" (Angrist and Pischke, 2009)

- Randomized control trial
- Selection on observables (unconfoundedness)
- Instrumental variables (LATE and Marginal Treatment Effects setups)
- **Difference-in-Differences (DiD)**
- Regression discontinuity designs

Popularity of Difference-in-Differences methods

Currie et al. (2020) at AEA P&P

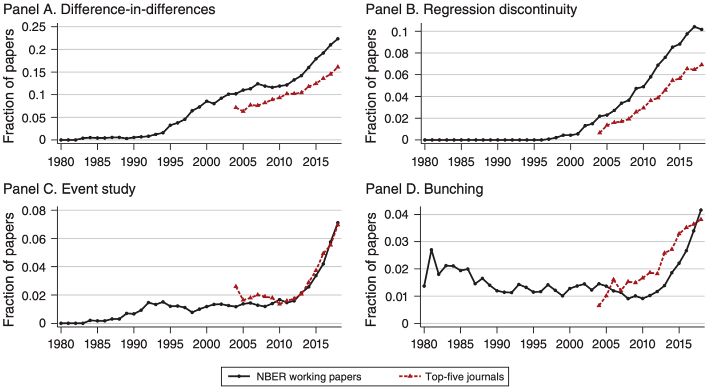

Figure 4. Quasi-experimental Methods

Notes: This figure shows the fraction of papers referring to each type of quasi-experimental approach. See Table A.I for a list of terms. The series show five-year moving averages.

What is the appeal of

Difference-in-Difference methods?

The appeal of Difference-in-Differences

- DiD methods exploit variation in time (before vs. after) and across groups (treated vs. untreated) to recover causal effects of interest.
- **Pre vs. Post comparisons**
  - **Compares:** same individuals/communities/groups of units before and after program.
  - **Limitation:** Does not account for potential trends in outcomes.
- **Treated vs. Untreated comparisons**
  - **Compares:** participants to those who have not experienced treatment (at least not yet).
  - **Limitation:** Selection – is participation driven by other factors?
- **DiD combines these two approaches to avoid their pitfalls.**

The Canonical Difference-in-Differences estimator

- **The canonical DiD estimator** is given by

$$\widehat{\theta}_n^{DiD} = \left(\bar{Y}_{g=treated,t=post} - \bar{Y}_{g=treated,t=pre}\right) - \left(\bar{Y}_{g=untreated,t=post} - \bar{Y}_{g=untreated,t=pre}\right),$$

where $\bar{Y}_{g=d,t=j}$ is the sample mean of the outcome Y for units in group d in time period j,

$$\bar{Y}_{g=d,t=j} = \frac{1}{N_{g=d,t=j}} \sum_{i=1}^{N_{all}} Y_i 1\{G_i = d\} 1\{T_i = j\},$$

with

$$N_{g=d,t=j} = \sum_{i=1}^{N_{all}} 1\{G_i = d\} 1\{T_i = j\},$$

G_i and T_i are group and time dummy, respectively, and Y_i is the "pooled" outcome data.

Some DiD Examples

- Card and Krueger (1994): Effect of minimum wage on employment.
  - Compared the changes in wages, employment, and prices at stores in New Jersey (increased minimum wage) relative to stores in Pennsylvania (minimum wage remained fixed).
- Meyer, Viscusi and Durbin (1995): Effect of weekly benefit amount on time out of work due to injury.
  - Compared high-earnings (affected by the policy change) and low-earnings (not affected by the policy change) individuals injured before and after increases in the maximum weekly benefit amount. Estimated effects in Kentucky and Michigan.
- Malesky, Nguyen and Tran (2014): Effect of government recentralization in Vietnam on public services.
  - Compared provinces (and districts) that abolished elected councils in Vietnam to other provinces who did not abolish, before and after the recentralization. Analyzed 30 outcomes.

Some DiD Examples

- Venkataramani, Shah, O'Brien, Kawachi and Tsai (2017): Effect of US Deferred Action for Childhood Arrivals (DACA) immigration program on health outcomes.
    - Compared changes in health outcomes among individuals who met key DACA eligibility criteria (based on age at immigration and at the time of policy implementation) before and after program implementation versus changes in outcomes for individuals who did not meet these criteria.
- Venkataramani and Chatterjee (2019): Effect of early Medicaid expansions and drug overdose mortality in the USA.
    - Compared changes in drug overdose mortality in early expansion states versus non-expansion states.

Some DiD Examples

- Cunningham and Shah (2018): Effect of decriminalization of indoor prostitution on composition of the sex market, reported rape offences, and sexually transmitted infections.
  - Compared outcomes in Rhode Island (decriminalized indoor sex work in July 2003) with other states, before and after the decriminalization. Analyzed the effect on many outcomes.
- Benzarti and Carloni (2019): Effect of incidence of value-added taxes for French sit-down restaurants.
  - Compared sit-down restaurants (experienced a cut in value-added taxes) with other market services (not affected by the policy), before and after the tax cut. Analyzed the effect on prices, costs of labor an non-labor intermediate input, owner's profit, and employment.

Some DiD Examples

- Assunção, Gandour, Rocha and Rocha (2020): Effect of rural credit on deforestation
    - Compared municipalities within the Amazon biome (concession of subsidized rural credit for them are conditional on stricter requirements since 2008), with municipalities outside the border the Amazon biome (not affected by the policy change), before and after the policy.

But what kind of causal effect

parameter $\widehat{\theta}_n^{DiD}$ is actually recovering?

What kind of identification assumptions do we need to impose to attach a causal interpretation to $\widehat{\theta}_n^{DiD}$?

Potential Outcomes

Causality with Potential Outcomes

- We will adopt the **Rubin Causal Model** and define potential outcomes.
- Potential outcomes will reflect time you are first-treated (we can “play” with this later).
- Let $Y_{i,t}(g)$ be the potential outcome for unit i, at time t, if this unit is first treated at time period g.
- T periods: $t = 1, ..., T$.
- Let $G_i \in \mathcal{G} \subset \{1, ..., T\} \cup \{\infty\}$ denote the time unit i is first-treated, with the notion that if a unit is “never-treated”, $G_i = \infty$.
- Observed outcome data in time period t for unit i is given by $Y_{i,t} = \sum_{g \in \mathcal{G}} 1\{G_i = g\} Y_{i,t}(g)$.

Causality with Potential Outcomes - The "never treated" group

- We are calling a group "never treated" if this set of units remains untreated in all time periods in our data.
- With two time periods $t = 1, 2$, we call the group of units that are still not exposed to treatment by time $t = 2$ the "never treated".
  - This is the case even if some of these units are eventually treated at time $t = 3$ (which we do not have access to this data yet).
- This is an abuse of notation, but can help us with intuition.

Causality with Potential Outcomes in the canonical 2x2 DiD setup

- Let's focus on the **Canonical 2x2 setup**.
- There are n units available, $i = 1, 2, \ldots, n$.
- There are two time periods available, $t = 1$ and $t = 2$.
- A subset of all units are treated at time $g = 2$ (treated units), and a subset of units remain untreated at time $t = 2$, so $\mathcal{G} = \{2, \infty\}$.
- For units that are treated in time period $g = 2$, we observe $Y_{i,t=1}(2)$ and $Y_{i,t=2}(2)$.
- For the "never treated" units $g = \infty$, we observe $Y_{i,t=1}(\infty)$ and $Y_{i,t=2}(\infty)$.

Causality with Potential Outcomes in the canonical 2x2 DiD setup

- **Treatment Effect**
  - The treatment effect or causal effect of the treatment on the outcome of unit i at time t is the difference between its two potential outcomes:

$$Y_{i,t}(2) - Y_{i,t}(\infty)$$

- **Observed outcome**
  - Observed outcomes at time t are realized as

$$Y_{i,t} = 1\{G_i = 2\}Y_{i,t}(2) + 1\{G_i = \infty\}Y_{i,t}(\infty).$$

- **Fundamental problem of causal inference**
  - At time t we cannot observe both potential outcomes $Y_{i,t}(2)$ and $Y_{i,t}(\infty)$.

Fundamental problem of causal inference: Missing data problem

	Data				
Unit	$Y_{i,t=1}(2)$	$Y_{i,t=2}(2)$	$Y_{i,t=1}(\infty)$	$Y_{i,t=2}(\infty)$	G_i
1	?	?	✓	✓	∞
2	✓	✓	?	?	2
3	?	?	✓	✓	∞
4	✓	✓	?	?	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	✓	✓	?	?	2

✓: Observed data

?: Missing data (unobserved counterfactuals)

Causality with Potential Outcomes in the canonical 2x2 DiD setup

- **Problem:**
  - Causal inference is difficult because it involves missing data.
  - At time t, how can we find $Y_{i,t}(2) - Y_{i,t}(\infty)$?
- **"Cheap" solution - Rule out heterogeneity.**
  - $Y_{i,t}(2), Y_{i,t}(\infty)$ constant across units.
  - $Y_{i,t}(2), Y_{i,t}(\infty)$ constant across time periods and impose a no-anticipation assumption (more on this in a bit).
- **But Causal inference is all about heterogeneity.**
  - In these cases, the "cheap solution" doesn't work and we need to find other paths.
  - We need to find more appealing assumptions!

Let’s first be explicit about a “hidden” assumption embedded in our analysis

SUTVA and No-Anticipation Assumption

Stable Unit Treatment Value Assumption (SUTVA)

Assumption (SUTVA)

Observed outcomes at time t are realized as

$$Y_{i,t} = \sum_{g \in \mathcal{G}} 1\{G_i = g\} Y_{i,t}(g).$$

- In the 2x2 DiD case, observed outcomes at time t are realized as

$$Y_{i,t} = 1\{G_i = 2\} Y_{i,t}(2) + 1\{G_i = \infty\} Y_{i,t}(\infty).$$

Stable Unit Treatment Value Assumption (SUTVA)

Assumption (SUTVA)

Observed outcomes at time t are realized as

$$Y_{i,t} = \sum_{g \in \mathcal{G}} 1\{G_i = g\} Y_{i,t}(g).$$

- Implicitly implies that potential outcomes for unit i are not affected by treatment of unit j.
    - Rules out interference across units
    - Rules out spillover effects
    - Rules out general equilibrium effects

Stable Unit Treatment Value Assumption (SUTVA)

Assumption (SUTVA)

Observed outcomes at time t are realized as

$$Y_{i,t} = \sum_{g \in \mathcal{G}} 1\{G_i = g\} Y_{i,t}(g).$$

- This assumption may be problematic in some applications
- **We should choose the units of analysis to minimize interference across units.**