STATISTICAL HORIZONS

Latent Growth Curve Modeling

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Upcoming Seminar:

June 1-2, 2017, Philadelphia, Pennsylvania

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LATENT GROWTH CURVE MODELING

TOPICS

Review of SEM and Software Basics
Mean Structure Models
Linear Model Foundations
Nonlinear Models
Other Cool Stuff
Sample Size Planning

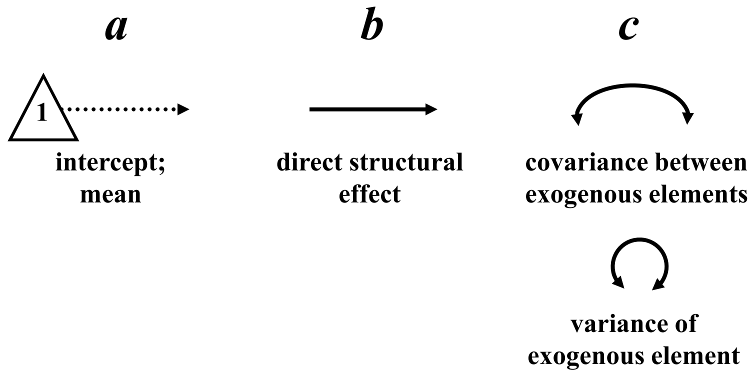
May 20-21, 2016
Philadelphia, PA

REVIEW OF SEM AND SOFTWARE BASICS

Classic Notational Systems and Conventions

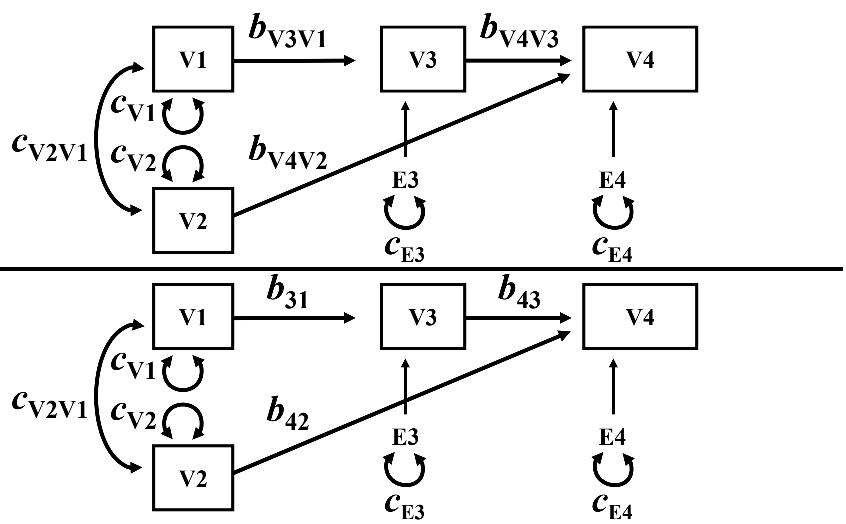
Bentler-Weeks (BW)		**Jöreskog-Keesling-Wiley (JKW)** ("LISREL" notation)	
V	**observed Variable**		x y
F	**latent Factor**	Exogenous: Endogenous:	ξ (ksi) η (eta)
E	**Error (in Vs)**		δ (delta) ε (epsilon)
D	**Disturbance (in Fs)**		ζ (zeta)

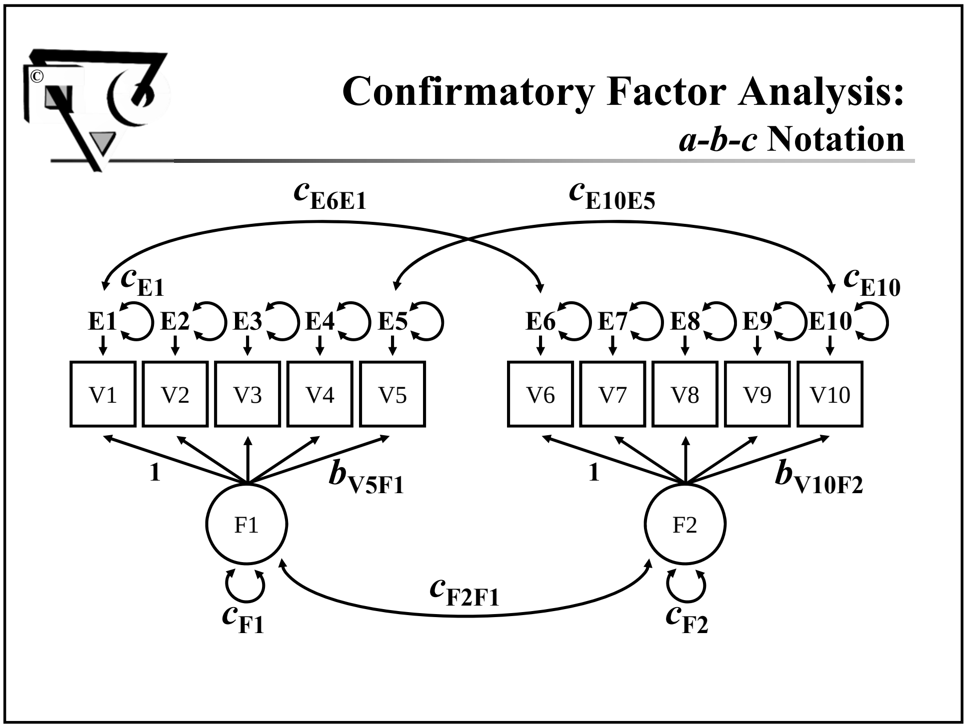
Parameter Notation System:
Hancock-Mueller *a-b-c* Parameter Notation*

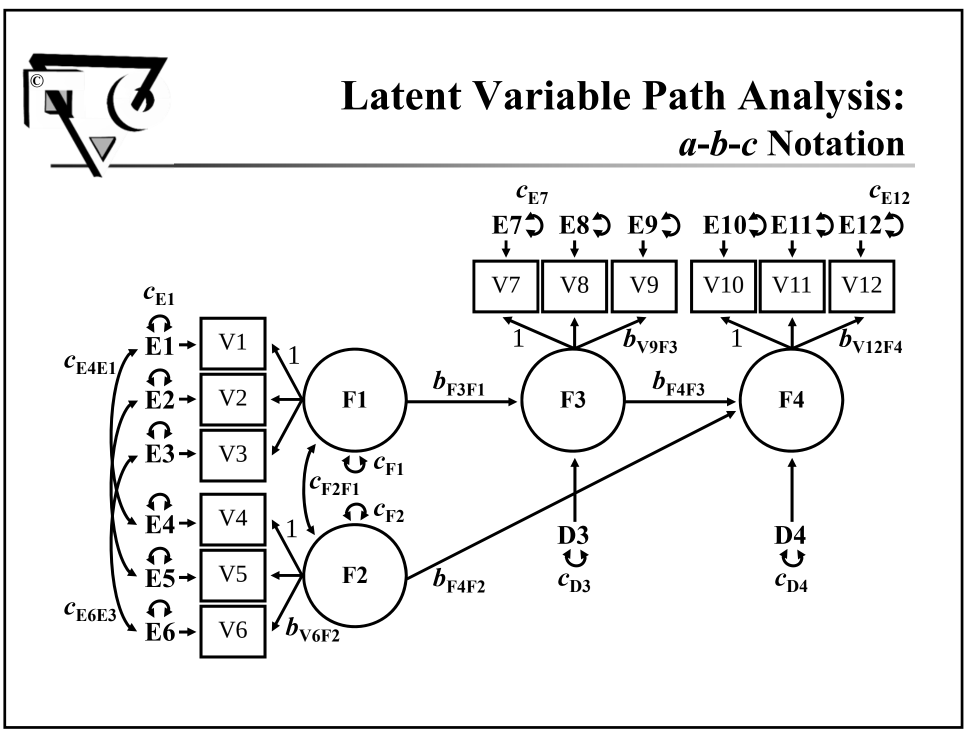


* Hancock & Mueller, 2006/2013

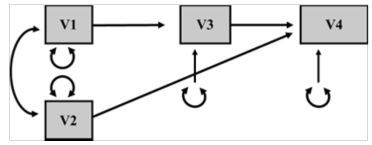
Measured Variable Path Analysis:
a-b-c Notation




©
Confirmatory Factor Analysis:
a-b-c Notation
cE6E1
cE10E5
cE1
cE10
E1 E2 E3 E4 E5
E6 E7 E8 E9 E10
V1 V2 V3 V4 V5
V6 V7 V8 V9 V10
1
bV5F1
1
bV10F2
F1
F2
cF2F1
cF1
cF2

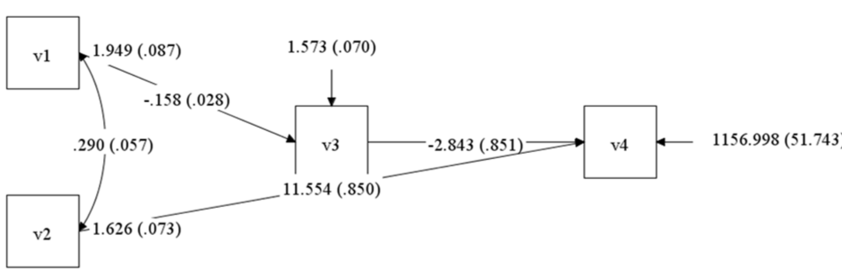

©
Latent Variable Path Analysis:
a-b-c Notation
cE7
cE12
E7 E8 E9 E10 E11 E12
V7 V8 V9 V10 V11 V12
1
bV9F3
1
bV12F4
cE1
cE4E1
E1 E2 E3 E4 E5 E6
V1 V2 V3 V4 V5 V6
1
F1
bF3F1
F3
bF4F3
F4
cF1
cF2F1
cF2
1
F2
bF4F2
bV6F2
cE6E3
D3
cD3
D4
cD4

Measured Variable Path Analysis:
Mplus syntax




```
DATA:
  FILE IS mvpa_data.csv;
VARIABLE:
  NAMES ARE V1 V2 V3 V4;
ANALYSIS:
  ESTIMATOR IS ML;     } default
MODEL:
  V3 ON V1;
  V4 ON V2 V3;
  V1; V2; V3; V4;      } default
  V1 WITH V2;
OUTPUT:
  SAMPSTAT STANDARDIZED;
```

Measured Variable Path Analysis:
Mplus output

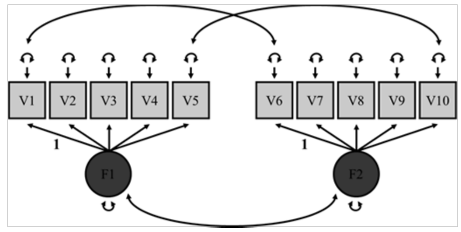


Confirmatory Factor Analysis:
Mplus syntax

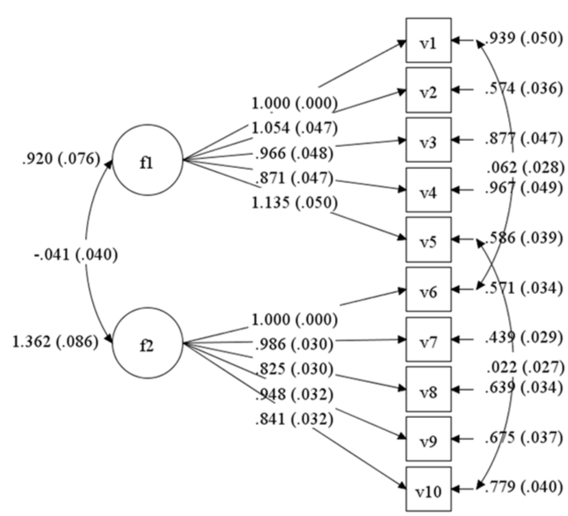
```
DATA:
  FILE IS cfa_data.csv;
VARIABLE:
  NAMES ARE V1-V10;
MODEL:
  F1 BY V1-V5;
  F2 BY V6-V10;
  V1 WITH V6;
  V5 WITH V10;
  V1-V10;
  F1-F2;
  F1 WITH F2;
OUTPUT:
  SAMPSTAT STANDARDIZED;
```

F1 BY V1-V5; F2 BY V6-V10; — first loading set to 1 by default

V1-V10; F1-F2; F1 WITH F2; — default

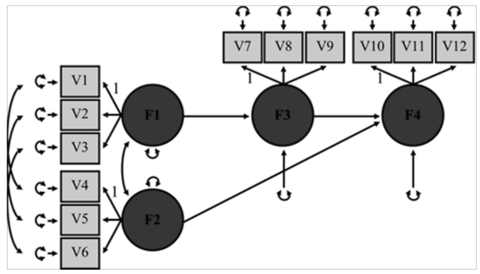


Confirmatory Factor Analysis:
Mplus output

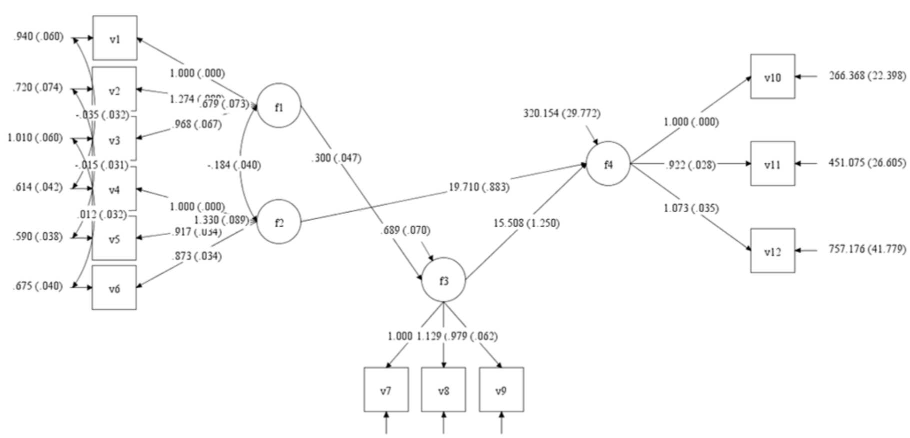


Latent Variable Path Analysis:
Mplus syntax

```
DATA:
  FILE IS lvpa_data.txt;
VARIABLE:
  NAMES ARE V1-V12;
MODEL:
  F1 BY V1-V3;
  F2 BY V4-V6;
  F3 BY V7-V9;
  F4 BY V10-V12;
  V1-V3 PWITH V4-V6;
  F3 ON F1;
  F4 ON F2 F3;
  V1-V12; F1-F4;  } default
  F1 WITH F2;     }
OUTPUT:
  SAMPSTAT STANDARDIZED;
```



Latent Variable Path Analysis:
Mplus output



Select Data-Model Fit Indices

Absolute (observed vs. model-implied var/cov matrix)	**Parsimonious** (adjust for model complexity)	**Incremental** (target vs. baseline model)
Model χ^2 statistic	**Akaike Information Criterion (AIC)**	**Comparative Fit Index (CFI)** $\geq$ .95*
Standardized Root Mean Squared Residual (SRMR) $\leq$ .08*	**Root Mean Squared Error of Approximation (RMSEA)** $\leq$ .06*	**Normed Fit Index (NFI)**
Goodness-of-Fit Index (GFI)	**Adjusted Goodness-of-Fit Index (AGFI)**	**Nonnormed Fit Index (NNFI; also known as Tucker-Lewis Index)**

* Hu & Bentler (1999)

Data-Model Fit Assessment: Interpretation

- Poor data-model fit?

  Reject the hypothesized model. Entertain modifications *only* if they make theoretical and statistical sense.

- Satisfactory data-model fit?

  Tentatively retain the proposed model as *one* viable representation of the true relations underlying the data

SEM stuff you should already know

- Measured variable path models
- Confirmatory factor models
- Latent variable path models (general SEM)
- Multisample covariance structure models
- Model / parameter identification
- Estimation
- Data-model fit assessment
- Model modification/respecification
- SEM software (e.g., AMOS, EQS, LISREL, Mplus, lavaan)

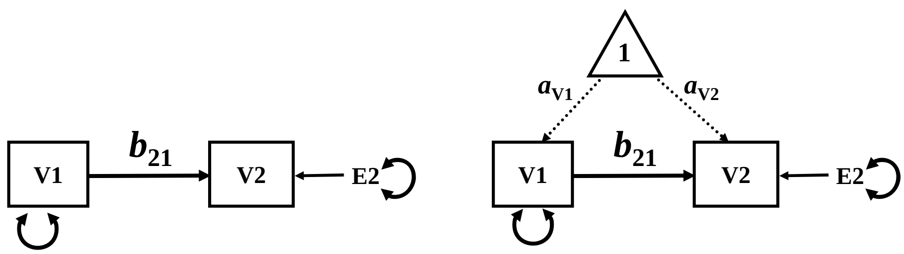
Select Introductory SEM Texts

- Beaujean, A. A. (2014). *Latent variable modeling using R*. New York: Taylor & Francis.
- Bollen, K. A. (1989). *Structural equations with latent variables*. New York: Wiley.
- Byrne, B. M. (2012). *Structural equation modeling with Mplus*. New York: Taylor & Francis.
- Finch, W. H., & French, B. F. (2015). *Latent variable modeling with R*. New York: Routledge.
- Hayduk, L. A. (1987). *Structural equation modeling with LISREL*. Baltimore, MD: The Johns Hopkins University Press.
- Kelloway, E. K. (2015). *Using Mplus for structural equation modeling* (2nd ed.). Thousand Oaks, CA: Sage.
- Kline, R. B. (2016). *Principles and practice of structural equation modeling* (4th ed.). New York: The Guilford Press.
- Loehlin, J. C. (2004). *Latent variable models* (4th Ed.). Hillsdale, NJ: Erlbaum.
- Raykov, T., & Marcoulides, G. A. (2006). *A first course in structural equation modeling* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum.
- Schumacker, R. E., & Lomax, R. G. (2016). *A beginner's guide to structural equation modeling* (4th ed.). New York: Routledge.

MEAN STRUCTURE MODELS

- intercepts
- repeated measures
- latent means
- latent repeated measures
- limitations

Measured Mean Structure:
A simple regression example



$V2 = b_{21}V1 + E2$

"structural equation"

$V2 = a_{V2}(1) + b_{21}V1 + E2$

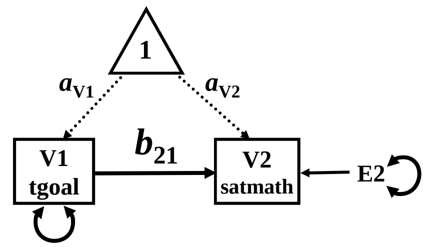
$V2 = a_{V2} + b_{21}V1 + E2$

a_{V2} is an intercept term

a_{V1} is a mean

Measured Mean Structure:
Mplus Syntax

```
DATA:
  FILE IS simple_data.csv;
VARIABLE:
  NAMES ARE tgoal satmath;
MODEL:
  satmath ON tgoal;
  tgoal; satmath;          } default
  [tgoal]; [satmath];      }
OUTPUT:
  SAMPSTAT STANDARDIZED;
```



Data from n=1000 9th grade girls on task goal orientation (tgoal) and Stanford Achievement Test math score (satmath).

Measured Mean Structure:
Mplus Output

```
MODEL FIT INFORMATION

Chi-Square Test of Model Fit

          Value                              0.000
          Degrees of Freedom                     0
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.000
          90 Percent C.I.                    0.000  0.000
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                1.000 }
          TLI                                1.000 }

SRMR (Standardized Root Mean Square Residual)

          Value                              0.000
```

With mean stuctures, incremental fit indices are generally ill-advised unless a null model is computed separately and comparative indices are hand-derived.

Measured Mean Structure:
Mplus Output

```
                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 SATMATH  ON
    TGOAL             -1.428      0.920     -1.553      0.120

 Means
    TGOAL              3.308      0.040     82.160      0.000

 Intercepts
    SATMATH          690.063      3.259    211.711      0.000

 Variances
    TGOAL              1.621      0.072     22.361      0.000

 Residual Variances
    SATMATH         1370.822     61.307     22.360      0.000


STDYX Standardization
                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 SATMATH  ON
    TGOAL             -0.049      0.032     -1.555      0.120
```

Measured Mean Structure:
Mplus Output / SPSS Output

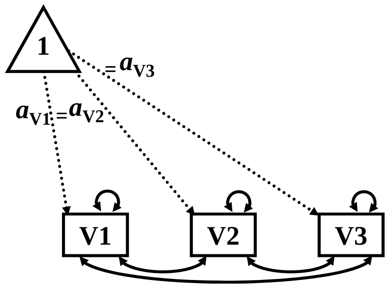
```
                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 SATMATH  ON
    TGOAL             -1.428      0.920     -1.553      0.120

 Intercepts
    SATMATH          690.063      3.259    211.711      0.000

STDYX Standardization
                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 SATMATH  ON
    TGOAL             -0.049      0.032     -1.555      0.120
```

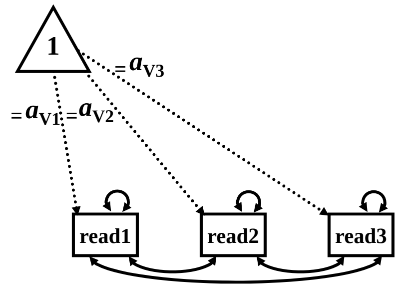
Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	690.063	3.263		211.503	.000
	tgoal	-1.428	.920	-.049	-1.551	.121

a. Dependent Variable: satmath

Measured Mean Structure:
Repeated Measure Designs, Measured



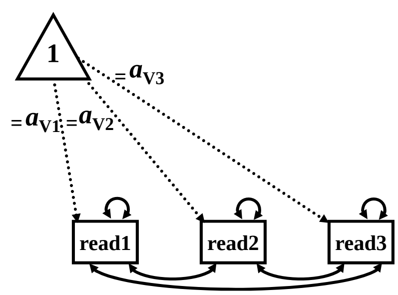


The χ^2 for the above model, with intercept constraints, corresponds to an omnibus repeated measures test, but without requiring the assumption of sphericity.

Data from n=86 Korean adults learning to read English across 3 months.

Measured Mean Structure:
Mplus Syntax

```
DATA:
  FILE IS repeated_data.txt;
VARIABLE:
  NAMES ARE read1-read3;
MODEL:
  read1 WITH read2-read3;
  read2 WITH read3;
  read1-read3;
  [read1] (a);
  [read2] (a);
  [read3] (a);
OUTPUT:
  SAMPSTAT STANDARDIZED;
```



Data from n=86 Korean adults learning to read English across 3 months.

Measured Mean Structure:
Mplus Output

```
Chi-Square Test of Model Fit
          Value                              13.679
          Degrees of Freedom                      2
          P-Value                            0.0011

                                                 Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 READ1    WITH
    READ2             31.859      5.228      6.094      0.000
    READ3             29.457      4.946      5.956      0.000

 READ2    WITH
    READ3             28.917      4.627      6.249      0.000

 Means
    READ1             20.419      0.617     33.113      0.000
    READ2             20.419      0.617     33.113      0.000
    READ3             20.419      0.617     33.113      0.000

 Variances
    READ1             42.900      6.372      6.732      0.000
    READ2             36.110      5.213      6.928      0.000
    READ3             33.723      4.878      6.913      0.000
```

Measured Mean Structure:
Mplus Output / SPSS Output

```
Chi-Square Test of Model Fit
          Value                              13.679
          Degrees of Freedom                      2
          P-Value                            0.0011

                                                 Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Means
    READ1             20.419      0.617     33.113      0.000
    READ2             20.419      0.617     33.113      0.000
    READ3             20.419      0.617     33.113      0.000
```

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
time	Sphericity Assumed	116.694	2	58.347	8.377	.000
	Greenhouse-Geisser	116.694	1.955	59.694	8.377	.000
	Huynh-Feldt	116.694	1.995	58.480	8.377	.000
	Lower-bound	116.694	1.000	116.694	8.377	.005
Error(time)	Sphericity Assumed	1323.306	190	6.965		
	Greenhouse-Geisser	1323.306	185.714	7.125		
	Huynh-Feldt	1323.306	189.570	6.981		
	Lower-bound	1323.306	95.000	13.930		

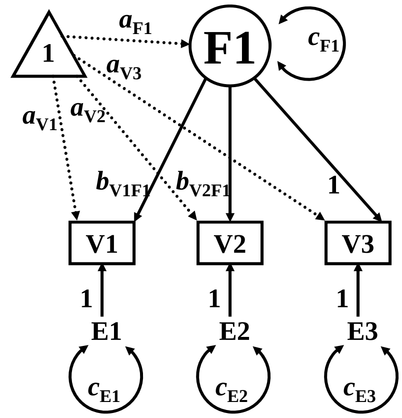
Latent Mean Structure:
Introduction

Structural equations

V1 = $a_{V1}(1) + b_{V1F1}F1 + E1$

V2 = $a_{V2}(1) + b_{V2F1}F1 + E2$

V3 = $a_{V3}(1) + 1\ F1 + E3$



a_{V1}, a_{V2}, a_{V3} are intercept terms

a_{F1} is a latent mean

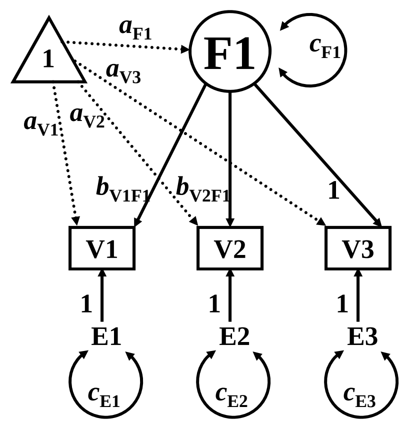
Latent Mean Structure:
Introduction

Model-Implied Covariance Matrix with Six Unknowns

$$\begin{bmatrix} \underline{b^2_{V1F1}}c_{F1} + \underline{c_{E1}} & & \\ b_{V1F1}\underline{b_{V2F1}}c_{F1} & b^2_{V2F1}c_{F1} + \underline{c_{E2}} & \\ b_{V1F1}c_{F1} & b_{V2F1}\underline{c_{F1}} & c_{F1} + \underline{c_{E3}} \end{bmatrix}$$

Model-Implied Mean Vector with Four Additional Unknowns

$$\begin{bmatrix} \underline{a_{V1}} + b_{V1F1}\underline{a_{F1}} & \underline{a_{V2}} + b_{V2F1}a_{F1} & \underline{a_{V3}} + a_{F1} \end{bmatrix}$$



The mean structure is currently under-identified.